M316K – Foundations of Arithmetic

Spring 2009

Project 1

Instructions and Guidelines

1. This project consists of several mathematical challenges; each challenge has the point value indicated. Your group should choose at least one hundred points' worth of challenges to complete. You may do more than a hundred points' worth of challenges if you wish, but your score will be adjusted so that it is out of a hundred points. (However, I may make some adjustments in your favor if you do a significant amount of extra work and your grade doesn't reflect this.)

2. Your group must work on each challenge *together*; you are not allowed to assign challenges to various group members and have one group member complete one challenge on his/her own, another group member complete another challenge on his/her own, and so on. However, you will need to appoint a group member as a "scribe" for each challenge; this group member is responsible for any writing that needs to be done for the challenge, though other group members are allowed to review and make suggestions regarding the scribe's work. Each group member must scribe for at least one challenge.

3. Each member of the group is responsible for everything that the group does. You should be familiar with all of the work that your group turns in, to the point that you would be able to explain and/or defend it to somebody else.

4. Please cite any books, notes, websites, or other resources you use to complete this project; do this at the end of your group's submission for each challenge. I'm not picky about citation styles and formatting, but it is very important for you to give credit to any source you receive information from, even if this information is not quoted in your group's final work.

5. If you anticipate that you will not be able to work with your group on this project, because of conflicts of schedule, personality, or otherwise, please let me know **immediately** and I will either reassign you to another group or assign you an individual project. (I will assign an individual project only as a last resort, and if I do, it won't be any less work for you than the group project would be.) Under no circumstances should you let this matter slide without telling me and try to claim credit for work you didn't do; it won't work.

6. **Scoring:** I will grade your group's submission for each challenge on a scale of zero to N points, where N is the number of points assigned to the challenge. However, work that goes above and beyond the call of duty may earn more than the "maximum" number of points. Everyone in your group will earn the same grade on this project, provided that I determine (and the group agrees) that everyone contributed equally to the work on all parts of the project.

7. The project is due in class on **Monday, March 30**; no exceptions. Make arrangements so that unexpected circumstances won't keep you from turning in the project on this day.

Tips and Suggestions

8. **Start working early.** If you wait until the last day (or even the last week) to do the entire project, you will not produce work of the quality I am expecting. I strongly recommend that you start working this week so that you can get a feel for the issues and difficulties that will come up. This will also help protect you in case it turns out that your group cannot work together due to scheduling difficulties.

9. Use technology to communicate; if you use e-mail and instant messaging well, you won't have to meet in person quite as often. I do recommend that you meet in person at least once a week to touch base and discuss what's going on, but that's a suggestion, not a rule.

10. Keep in mind that Spring Break will eat up a week in the middle of your work time for this project. It's very likely that at least one of your teammates will be out of town for that week, so don't count on being able to get everybody together during that time. If you work efficiently and don't procrastinate, you will be able to complete the project without doing any work over the break.

11. If you have any questions about the challenges, or want to get some feedback from me on what you're doing, please feel free to visit me during office hours. I won't give you much advice on how to complete the challenges, because I want you to figure that out as a group, but I am happy to clear up anything that may be confusing about the tasks. And I am definitely willing to look at your work and give you my honest (but non-binding) opinion.

A Note on Academic Integrity

The concept of academic integrity is very important in any educational setting, and it is especially important to this project. I have chosen mathematical tasks that I hope you will find worthwhile and interesting to work on. It may be possible to short-cut some parts of this project by plagiarizing books or websites, but I strongly warn you not to do so. Plagiarism includes, but is not limited to, quoting part of a book or website without crediting the author, borrowing ideas or knowledge from another source without giving proper credit (even if these ideas or knowledge are stated in words other than the original author's), or writing a report that is basically a recycled or rephrased version of somebody else's work without adding your own ideas or analysis (even if you do credit the source). In the case of this particular project, plagiarism may also consist of claiming that a solution to a problem is original when in fact you retrieved it (or a solution to a similar problem) from the internet. For more information, please see http://www.plagiarism.org/learning_center/what_is_plagiarism.html.

I don't want to dwell on this issue very much, because I genuinely like my students and feel like I have always been blessed with very good classes of students. Sadly, even in a very good class, there may be one or two students who would rather try to cheat than make an honest effort. I'm usually a very nice guy (most of you probably know this by now), but that's partly because I save all my mean-spiritedness for cheaters. So please don't put yourself in a situation where you're likely to see my mean side.

Challenge 1: Raise Your Flag – 30 points

A few weeks ago, when I created your study groups, I named your group after a mathematician. The purpose of this challenge is for you to get to know your group's mathematician, the work that he[1] did, and the time in which he lived.

1. Learn as much as you can about your team's mathematician – his life, his work, and the time and place in which he lived. You can probably get a lot of good information from Wikipedia, but in many cases you will probably get much more interesting anecdotal information if you try other sources.

2. Design and create a team flag. This flag should be larger than a standard sheet of paper (I'll leave it to you to decide the exact size and shape of the flag), and it should be on a material that is sturdier than paper. Cloth or a similar material is best since most flags are printed on cloth, but I'll accept a flag on some other material such as poster board. **However, if your flag is too large to fit in my backpack and cannot be folded, I will need someone from your group to bring it to my office.**

   Your flag should satisfy the following requirements:

   (a) The flag *must* incorporate, as part of its design or graphics, your team's knowledge of the mathematician's mathematical work. This knowledge should be represented in a fairly specific way; for example, if your mathematician was an astronomer, it's not enough to put stars on the flag and call it a day. The flag should, in some sense, "tell a story" about what your mathematician discovered about those stars.

   (b) You should also try to incorporate some knowledge about your mathematician's life, or the time and place in which he lived (or both). This isn't as important as requirement (a); if you do a great job on requirement (a), I will be willing to forgive a lack of attention to this requirement.

   (c) The flag should be well designed. Most flags are not haphazardly assembled collections of images, so don't draw a bunch of loosely related pictures and call it a flag. You are absolutely encouraged to incorporate several different images or design elements, but you should try to put them together in a meaningful way.

   (d) Your flag doesn't have to bear the name of your mathematician, but I should be able to look at it and tell, without a doubt, which mathematician the flag is for.

   (e) The flag should reflect a good artistic effort by *all* of your team members.

3. Write a short essay (a page or two will suffice) about the design and imagery of your flag. What story does your flag tell? What theorem or discovery inspired your design? Did you incorporate elements of the mathematician's life and/or the setting in which he lived?

[1] Sadly, none of the sixteen mathematicians I chose were women. As a matter of fact there are many prominent women mathematicians today; some of them work in our math department! However, I wanted to choose mathematicians whose significance you could understand without having to do extensive research, and in order to do that, I had to choose mostly mathematicians from before the twentieth century. Those times were much less hospitable for women mathematicians, although there were some female superstars in the discipline even then. Hypatia (an ancient Greek) and Emmy Noether (a German algebraist) are two examples.

Challenge 2: Comparing Elementary Math Textbooks – 15 points

Elementary mathematics textbooks are at the front lines of the debate over what math we should teach young students, and how and when we should teach it. The purpose of this challenge is for you to get reacquainted with elementary math textbooks (which you probably haven't had much contact with in about ten years), and evaluate their usefulness as instructional materials.

For this challenge, you'll need to get your hands on two different elementary math textbooks. You should be able to find some of these books in the university's library system, but you'll have to do some poking around to figure out exactly where they are. If you are in contact with people who are currently teaching elementary school, you might want to see if they have any spare textbooks you can borrow.

1. As a group, pick out two different elementary math textbooks that you will analyze in this challenge. The books should be designed for the same grade level, but by different publishers. Be sure to write down which two textbooks you are using; provide enough information (author, title, publisher, date) so that I can track down a book if I need to do so.

2. First take some time to get acquainted with the material covered in each book. Are there any notable differences between the topics covered in the two books? (That is, are there topics covered by one book but not the other?) Does one textbook cover topics in a different order than the other? If so, which order do you think is more appropriate for a class that you might teach?

3. For each book, answer the following questions: is the textbook aligned with NCTM standards? (The answer to this question will almost certainly be "yes" for any state-adopted text; most major math textbook publishers make a very big deal out of showing how neatly aligned to NCTM standards their books are.) In what ways does the textbook reinforce the NCTM process standards: problem solving, representations, reasoning and proof, communication, and connections? For each process standard, give a specific example of how the textbook attempts to engage that standard.

4. Select two different topics – ones treated in both textbooks – that you think might be difficult for students who are using the books. For each topic, compare and contrast the two textbooks' treatments of the topic. Which book's treatment of the topic do you think is superior, and why? (On this part, you should look at several different aspects of the books: body text, examples, exercises, projects, and other material associated with the topic.)

5. Suppose that your team is in charge of selecting math textbooks for a school district. Out of these two textbooks, which one would you recommend, and why? (Assume that cost is not an issue.)

Notes

Please provide neatly written responses to all of the questions above. You should discuss all of these questions as a group, and only after everyone has had time to take an unbiased look at the textbooks your group is using. Keep in mind that I do not expect you to be math education experts, nor do I expect you to be fluent in math education jargon. I'm not looking for a great deal of polish on this challenge; what I do want is for you to have the experience of engaging in a group discussion where you debate the relative merits of several textbooks, and come to a consensus based on how well you think each textbook would serve your students.

Challenge 3: Some Texas-Sized Numbers – 25 points

In this challenge, you'll solve several problems in which you have to make reasonable estimates and deal with large quantities. For each problem, you'll probably need to do some "research" in order to figure out what assumptions you need to make. Feel free to use creative research methods: polling people, measuring objects and places featured in the problems, and trying things out for yourself. You may also use the internet to obtain data for this problem, but preferential treatment will be given to solutions that obtain information in direct, creative ways. If you do obtain data from the internet, please cite your sources.

When writing your solutions, please explain where *all* of the data and assumptions in your solutions come from. If they're blind estimates, say so. If you got them off the internet, say where you found them. If you used some other creative approach to get the data, explain what you did, and which member(s) of your group did it.

In this problem, you don't have to collect all of the data as a team; for example, if one person wants to go and measure birthday cakes for Problem 1, that's okay. But all team members should have a substantial share in the data-gathering process.

1. Suppose that we took all of the birthday cakes that people in the United States ate in the year 2008, and combined them into one giant birthday cake. Would this giant birthday cake fit inside Darrell K. Royal-Texas Memorial Stadium? (Obviously the stadium is not an enclosed building, so by "inside the stadium," I mean inside the stadium without rising above the highest bleachers in the stadium.)

2. What is the combined weight of all of the books in the Perry-Castaneda Library? Is this amount greater than a thousand tons? A million tons? A billion tons? (Please count only those books on shelves that are accessible to the general public; don't interrupt librarians' duties or students' studying in order to complete this problem. Also try to find a way to solve this problem without checking any books out. Part of the challenge here is to find a way to get the data you need without being disruptive or abusing the library's services.)

3. In a typical week (Monday through Friday), how many passengers do Capital Metro and UT Shuttle buses pick up at the bus stop on Guadalupe across from the Chipotle? I'm talking about the bus stop on the university side of the street, not the storefront. Be sure to account for daily fluctuations in bus ridership; you should expect the number of passengers picked up between 4 PM and 5 PM to be quite a bit greater than the number picked up between 9 AM and 10 AM. (Please feel free to make reasonable assumptions about the number of passengers picked up at night; I don't want you to feel like you have to hang out at a bus stop by yourself at night. Also, if you happen to see your friendly instructor on one of your bus-spotting trips, be sure to say hello.)

4. Suppose the alumni of the McCombs School of Business decide to flaunt their collective wealth by papering all the walls of the business buildings (both CBA and GSB in the UT building lexicon) with dollar bills. Assuming they only paper the walls in the halls and public areas, not the classrooms, would they be able to do this using only taxpayer money if all of the money allocated by the American Recovery and Reinvestment Act were devoted to this one ridiculous project? If so, would they be able to paper the walls with hundred-dollar bills instead of dollar bills?

5. If you wish, your team may craft its own UT-themed "big numbers" problem and solve it in place of one of the four problems given above. (Bonus points if the problem is especially creative and/or clever.)

Notes

Remember, no matter what methods you use to get information for this challenge, please be safe, stay out of trouble, and try not to get in other people's way. If done correctly, this challenge will require a fair amount of work, but if you divide up the research work, hopefully no one team member will find it too much of a burden. (I *do* want you all to discuss how you will approach each problem together before you divide up the work, and also work together to answer the questions after you've done the research.)

Challenge 4: Survey Says . . . – 20 points

The goal of this challenge is to further explore sets and Venn diagrams in the context of polling. Suppose I want to answer a question such as

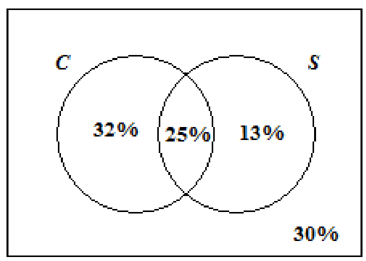
"Is a person more likely to go to college if he/she has an older sibling who has gone to college?"

One way to try to answer this question would be to ask a lot of people the following two questions:

1. Are you, or have you ever been, in college?
2. Do you have an older sibling who is or has been in college?

I might then let C be the set of people who go to college or have gone to college, and S the set of people who have older siblings who went to college. Based on the results of my poll, I could analyze the size of sets such as C, S, $C \cap S$, $C - S$, and so forth. This analysis would then allow me to make an educated guess about my original question. Answer the following questions:

1. Suppose that I enter the results of my poll in a Venn diagram as shown below. (The percentage entered in the region $C - S$ is the percentage of respondents in the set $C - S$, not the percentage in the set C.)



Based on the results of this poll, what would be your answer to the original question: is a person more likely to go to college if he/she has an older sibling who has gone to college? Justify your answer.

2. Explain why the poll does not answer this question with absolute certainty. Give as many reasons as possible.

3. Suppose you haven't seen the results of my poll, and that the results of my poll are not necessarily the ones shown in Problem 2. But you do know that I conducted the poll for this experiment on Facebook. Would this knowledge make you more confident or less confident in the scientific value of the poll? Explain. (*Hint:* The fact that Facebook is a social networking site and not a "scientific" or "serious" site may be relevant, but it's not the most important thing here.)

Now it's your turn to play the pollster. As a group, come up with three sets that you think might be related in an interesting way, but are not obviously guaranteed to be related. (An example of two sets that are obviously related is "the set of people who have played team sports in high school" and "the set of people who have worn shirts with numbers on them"; people in the former set are practically guaranteed to be in the latter. The idea is that the poll should give you a chance to discover a relationship that you suspected but weren't sure about.) My sets were "the set of people who have gone to college" and "the set of people with older siblings who have gone to college"; you should come up with *three* sets. The sets should be designed so that people will know with pretty good certainty whether they belong to them. A set like "the set of people who wear tennis shoes at least five days per week" is better than a set like "the set of people who wear tennis shoes regularly," because the latter is much more open to interpretation. Similarly, a set like "the set of people who are related to someone famous" is a terrible choice, not only because it's a matter of interpretation whether someone is famous or not, but because many people won't have enough genealogical information to be able to determine for sure whether they're in the set.

4. Define the three sets that your team came up with, and give each set a name. Try to use names that are somewhat suggestive; don't use the names A, B, and C unless you really think these letters are the best choice.

5. As a group, brainstorm **five** questions related to these three sets that you'd like to try to answer. The majority of these questions should involve relationships between at least two of the sets (in other words, don't limit yourself to one-dimensional questions like "do at least 20% of Americans smoke?").

6. As a group, decide how you will carry out the polling for this question. Think about how many people you want to poll, how you want to select people to poll, and how exactly you will pose the questions. (At this point you may find yourselves tweaking the definitions of your sets; this is okay. You want to make sure to set up the questions so that they are precise and likely to elicit truthful responses.) Write a few paragraphs about the issues that your group anticipated, and how you decided to deal with them.

7. Conduct the poll, and represent your results in a Venn diagram. Represent your results in the Venn diagram as percentages, but also indicate how many people you polled.

8. Use the results of your poll to answer the five questions your team decided to try to answer. Keep in mind that your answers to some of these questions may be along the lines of "the poll didn't give us enough information to determine the answer to this question." If this happens, explain why this is the case.

9. The nature of any poll is that it cannot be perfectly accurate (some recent presidential election polls have been rather infamous examples of this). Discuss some aspects of your team's polling process that might limit the accuracy of your poll (that is, the ability of the poll to answer questions about the entire population you're studying).

Notes

Keep in mind that certain topics (especially religion, politics, and socioeconomic status) are sensitive subjects for a lot of people. My hope is that your team can come up with some interesting questions that don't require you to question people about hot-button issues. If your team does want to try to explore questions about potentially controversial or personal subjects, I strongly recommend that you do the polling in as safe and unintrusive a manner as possible. The idea of using Facebook to poll people isn't necessarily a bad one (and it would probably be very easy to do), but keep in mind that a Facebook poll will be a very biased source of information on certain kinds of questions. Be judicious about how you do the polling; try to get good results, but don't put yourself or your team members in a position where you're likely to annoy or alienate people.

Challenge 5: Sideways Math – 20 points

When I was about ten years old, I discovered an amazing book titled *Sideways Arithmetic From Wayside School* by acclaimed children's author Louis Sachar. Louis Sachar has authored a number of books, most of them quite humorous, about the fictional Wayside School and the class taught by Ms. Jewls. Most of the books are just enjoyable children's novels, but *Sideways Arithmetic* doubles as a puzzle book. The "sideways arithmetic" puzzles featured in this book belong to a class of puzzles called "verbal arithmetic" or "alphametic" puzzles. In an alphametic puzzle, you are given an arithmetic fact in which the numbers are written as words, and your goal is to determine what digit each letter represents. Each instance of a letter in a puzzle stands for the same digit, and different letters are required to stand for different digits.

The most famous alphametic puzzle is the following addition puzzle posed by Henry Dudeney:

$$SEND + MORE = MONEY$$

In this puzzle, one can apply logical and arithmetic reasoning (and perhaps some brute-force guessing) to find that $O = 0$, $M = 1$, $Y = 2$, $E = 5$, $N = 6$, $D = 7$, $R = 8$, and $S = 9$. When these substitutions are applied, the alphametic equation becomes a correct addition statement:

$$9567 + 1085 = 10652$$

The best alphametic puzzles have unique solutions, as is the case with Dudeney's famous puzzle. (Notice that in this puzzle, the letters have to be assigned to the digits listed above; there is no other assignment of different digits to each letter that will make the addition statement correct.) However, it can still be interesting to think about alphametic puzzles that have no possible solutions, or ones that have only a handful of possible solutions. I have created several such puzzles for your enjoyment; and at the end of this challenge, you'll craft your own.

1. It is a well-known fact that one plus two is three. But when this statement is converted into alphametics, it becomes an impossible statement: $ONE + TWO = THREE$. Explain why this puzzle has no solutions.

2. One day, I was thinking about how they make the roads that we drive on. I figured maybe they just throw down some dirt and then lay tar on top of it, but after doing the alphametics, I decided that that couldn't possibly be right. Show that the puzzle $DIRT + TAR = ROAD$ has no solutions. (Keep in mind that each letter must stand for a different digit.)

3. It turns out that you can use alphametics to figure out some things about basic geometry, such as the fact that two lines make an angle. Find all possible solutions to the puzzle $LINE + LINE = ANGLE$. (Your answer to this question should be presented in as organized a manner as possible, and it should convince me that you have found *all* possible solutions.)

4. I've always thought it was funny that when you put the word "neck" in front of "lace," you get something that is not a piece of lace, but a piece of jewelry. Find all possible solutions to the puzzle $NECK + LACE = JEWEL$. (Again, be sure to convince me that you have found all possible solutions.)

5. You can also do alphametics with subtraction; for example, what do you get when you take all the smart people out of Texas? Just another state. Find all possible solutions to the puzzle $TEXAS - SMART = STATE$.

6. As a group, invent your own alphametic puzzle – one that has at least one solution – and find all possible solutions. You'll earn bonus points if your puzzle "makes sense" verbally like mine do and has a unique solution (or at least a very small number of solutions).

Notes
Remember, it is very important that you work as a group on this! I want everyone in your group to learn how to solve these puzzles, and have a hand in creating one.

Challenge 6: The Knights Who Say Nim – 20 points

The game of *Nim* is a game for two players. The game begins with one or more piles of stones. Different piles may contain different numbers of stones. Instead of drawing pictures of these piles of stones, I'll use the notation $(a_1, a_2, \ldots, a_m)$ to represent m piles of stones, with the first pile containing a_1 stones, the second pile containing a_2 stones, and so on. To make this more concrete, here are a few examples:

The notation $(1, 2, 3)$ represents three piles of stones, with the first pile containing just one stone, the second pile containing two stones, and the third pile containing three stones.

The notation (5) represents a single pile of five stones.

The notation $(4, 4, 4, 4)$ represents four piles, each containing four stones.

In this game, two players (whom we'll call Bonnie and Tony, with Bonnie always moving first) take turns removing stones from the piles. On each turn, a player is allowed to remove any number of stones from a single pile. The player is only allowed to remove stones from one pile. She/he must remove at least one stone. The player who removes the last stone is the winner (note that the player who wins may remove several stones on the winning move).

Here's a sample game. The configuration shown in each step is the configuration of stones after the player makes her/his move.

Start: $(4, 4, 4, 4)$
Bonnie: $(4, 1, 4, 4)$
Tony: $(4, 1, 4, 2)$
Bonnie: $(1, 4, 2)$ (Here Bonnie removes all of the stones in the first pile)
Tony: $(1, 1, 2)$
Bonnie: $(1, 1)$
Tony: (1)
Bonnie: WIN

In this challenge, we'll analyze the strategy of Nim. As part of this challenge, you will learn the winning strategy for Nim. You can either try to figure out the winning strategy yourselves, or you can look it up online. My hope is that you'll try to work out some of these problems on your own before looking up the winning strategy online, so that you can get a feel for the game. However, I don't have any objection to you looking up the strategy online – this is a good chance for you to practice dealing with online math resources, and separating the useful ones from the not-so-useful ones (of which there are many). I suggest that you try working out problems 1 through 5 without looking anything up, just to get a feel for the strategy. However, this is a suggestion, not a rule.

1. Suppose that the game starts with the configuration $(2, 2)$. Assuming that Bonnie and Tony both play as well as possible, who will win the game? Explain your answer.

2. Suppose that the game starts with the configuration $(3, 4)$. Assuming that Bonnie and Tony both play as well as possible, who will win the game? Explain your answer.

3. Suppose that the game starts with two piles of stones. Assuming that both players play as well as possible, who will win the game? (Your answer will depend on the number of stones in each pile.) Explain your answer.

4. Suppose that the game starts with the configuration $(3, 3, 3)$. Assuming that both players play as well as possible, who will win the game? Explain your answer.

5. Suppose that the game starts with the configuration $(1, 2, 3)$. Assuming that both players play as well as possible, who will win the game? Explain your answer.

The key to winning at Nim is to be able to look at a configuration of stones and determine whether it is a "winning" configuration or a "losing" configuration. We say that a configuration is a *winning* configuration if a player who receives this configuration will win the game if she/he plays as well as possible. (The configuration (5) is a winning configuration because a player who receives this configuration can simply win the game by removing all of the stones. The configuration $(1, 2)$ is also a winning configuration; if a player receives this configuration, she/he can win by removing one stone from the second pile.) We say that a configuration is a *losing* configuration if a player who receives this configuration will lose the game if both players play as well as possible. (The configuration $(1, 1)$ is a losing configuration.)

For problems 6 through 10, determine whether the configuration given is a winning configuration or a losing configuration. If it is a winning configuration, determine what move a player should make after receiving this configuration.

6. $(1, 3, 5)$

7. $(3, 4, 7)$

8. $(4, 4, 4, 4)$

9. $(12, 17, 19, 26)$

10. $(1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$

11. For the last part of this challenge, your job is to defeat your friendly (but very competitive) instructor at Nim. We'll play a series of seven games; you'll be required to win at least five. I'll decide the starting configuration for each game, but you will be allowed to choose whether to go first or second (which gives you complete control over the outcome). I'll let you take as much time as you need per turn, work together, and use scratch paper (which helps in this game); I will adhere to a strict one-minute time limit per turn and won't use scratch paper. Don't be intimidated by this requirement – if you really understand the strategy well, you will win every game handily.

Challenge 7: Math and Decision Making – 20 points

In Chapter 1, you worked on some problems in which you had to make some decisions based on considerations such as feasibility and cost-effectiveness (*e.g.* the problem of whether to hire a new employee, the calling card problem). The following problems are similar to those, except that in these problems, you will have to do more extensive research and write more thorough solutions. In the following problems, feel free to use the internet (or the real world!) to do whatever information-gathering you need to do.

1. Your friendly instructor's car is very old and needs to be retired. He plans to buy either a 2009 Toyota Corolla or a 2009 Toyota Prius (on that hefty graduate student salary, natch), and he needs your help in deciding which car to buy. Cody's primary concern is cost – both the cost of the car, and the cost of the fuel he will have to put into that car. Cody's work and personal life require him to drive an average of $10,000$ miles per year. About 75% of these miles are "highway" (high-speed) miles, and the other 25% of these miles are "city" (low-speed, stop-and-go) miles. He plans to use the car for ten years, then sell it and buy a new one. Help Cody decide which car to buy. Assume that Cody isn't interested in extra (non-standard) features, isn't worried about any possible difference in performance between the Corolla and the Prius, and isn't worried about other aspects (such as aesthetics or maintenance costs) of the cars. If the problem requires information (*i.e.* future fuel costs) that you can't pin down, discuss different scenarios that might occur, and determine whether different circumstances might warrant a different decision.

2. Suppose you are the head of a household containing four people – say, the members of your group. You want to feed your group for a week so that each group member gets 100% of his/her FDA-recommended daily calories and nutrients each day. You want to do this as cost-effectively as possible, while abiding by the following requirements:

1. **Number:** Your group should have three meals per day.
2. **Balance:** Your group should get all the required calories and nutrients without getting a highly excessive amount of the bad stuff: sugars, fats, sodium. (In other words, be wary of an all-Ramen diet.)
3. **Variety:** The meals should not be the same each day.

What is the cheapest menu you can come up with? Determine the cost of this menu as accurately as possible.

3. As a group, come up with your own problem in which you must make a real-life(ish) decision using a combination of research and basic math. Then solve this problem and report your results.

Notes

In this challenge, the most important thing I'm looking for is a well-written report for each problem that convinces me that (1) you've researched the options thoroughly and (2) the option you're recommending really is the best one. Even though you have to designate a scribe for this challenge, I expect you to work as a team when organizing your information and deciding what to say.

One more note: this challenge isn't just a silly exercise that I cooked up to put you to work. The sort of work you'll be doing here is the kind of work that consultants and actuaries are paid (often very well) to do on a regular basis!

Challenge 8: Fact Finder – 15–25 points (see note)

In this challenge, you'll solve a few logic problems, and invent two of your own. When you write your solutions to these problems, please be as organized as possible, detailing your thought process from start to finish. This will take a while if you do it properly, so don't leave the process of writing the explanation until the last minute!

1. Cody shows up for an exam review session to discover that the five students already there have decided to play a prank on him. Each of these students has been designated either as a "truth-teller", who makes only true statements, or a "liar", who makes only false statements. After chatting with his students for a few minutes, Cody realizes that not all of his students are being honest with him. Frustrated, he asks, "Isn't there anyone in here who won't lie to me?" The five students reply as follows:

Alicia: None of us are truth-tellers.
Brandin: Exactly one of us is a truth-teller.
Candyce: Exactly two of us are truth-tellers.
Denice: Exactly three of us are truth-tellers.
Emily: Exactly four of us are truth-tellers.

Based on these statements, help Cody figure out which students are telling the truth and which students are lying. Explain your reasoning.

2. After figuring out which of his students are telling the truth and which students are lying, Cody decides to join the fun, along with some other students who have arrived at the review. Cody's youngest student Kennan arrives at the review a few minutes late to find Cody and eight of his M316K students lined up on the stage. Kennan asks what is going on, and one of the students not on the stage (a truth-teller) explains to him that everyone on the stage is either a truth-teller or a liar. Kennan asks the people on stage, "Which of you are liars?" Rather than answering the question, the people on stage decide to tell Kennan a bit about themselves (or, as the case may be, not about themselves). They make their statements in order, from one end of the line to the other:

Brian: I learned Hapkido so that I could protect myself from gangs in El Paso.
Freddy: I play the accordion.
Sakeenah: I have a triplet who was adopted by another family.
Lesli: The first three toes on my feet are the same length.
Allison: I can play seven different musical instruments.
Ellen: I've been on television five times.
Jackie: I started dating a boyfriend because I prank-called him by dialing a random number.
Angelina: I am legally blind without contacts or glasses.
Cody: I once took – and passed – the Texas Bar Examination on a dare.

At this point, Kennan has a confused look on his face, and Marjorie (his mother) complains that the people on stage haven't given her and Kennan any way to figure out which people are truth-tellers and which are liars. So the people on stage decide to be a bit more helpful:

Brian: At least three of us are telling the truth.
Freddy: There are two liars standing next to each other.
Sakeenah: Angelina is a truth-teller.
Lesli: There are two truth-tellers standing next to each other.
Allison: The majority of the women in this line are liars.
Ellen: There are more female liars than male liars in this line.
Jackie: If we took Ellen out of the line, exactly half of the people in the line would be liars.
Angelina: One of the two guys at the front of the line is a truth-teller, and the other is a liar.
Cody: Lesli and I are both liars.

Help Kennan and Marjorie figure out which people are telling the truth and which are lying. (*Note:* You can, and should, solve this puzzle completely without using the "fun facts" (or lies) given above. I just added those so that you could have fun guessing which of those are true and which are lies.)

3. At the next review session, Cody decides to ask the first five students who show up for some biographical information. He asks each student where he/she grew up, and where he/she likes to eat in Austin. Unfortunately, by the end of the review session, Cody has forgotten which student grew up in which city, which student enjoys each of the restaurants he heard about, and in what order the students showed up. He does remember that the five students were Allison, Angelina, Denice, Rachael, and Romey; that the hometowns are Brea (California), Fort Worth (Texas), Green Bay (Wisconsin), San Antonio (Texas), and Troy (Michigan); and that the favorite restaurants are Hula Hut, Kenichi, Kyoto, Terra Burger, and Whole Foods. He also remembers that

1. The Californian's favorite place to eat is a Japanese restaurant.
2. The person who likes Whole Foods is from a state bordering the Great Lakes.
3. Angelina's favorite place to eat is part of a chain; besides having locations in Texas, this chain also has locations in her home state.
4. Romey arrived before Allison.
5. Denice is not from San Antonio.
6. The name of Romey's favorite restaurant contains the letter T.
7. Rachael arrived after the student from Fort Worth, but before the student from Green Bay.
8. The two people from Texas were first and fifth to arrive.
9. Romey's hometown is farther south than Rachael's.
10. The student who likes Kenichi arrived before the student who likes Whole Foods.
11. Denice's favorite restaurant is located on Guadalupe.
12. The two people who like Japanese food arrived consecutively.
13. Two of the female students are from Texas.
14. The student from Michigan did not arrive third.

Help Cody figure out each student's hometown and favorite restaurant, and the order in which the five students arrived.

4. As a group, create two original puzzles: one of the "truth-tellers and liars" variety (similar to Problem 2), and a logic grid puzzle (similar to Problem 3). Each puzzle must have a single correct solution that can be obtained from the clues using logic and, if necessary, careful guessing. You can look at other puzzles in order to get a better sense of what these puzzles look like, but the puzzles you create must be original, not based on already-existing puzzles. Be warned: creating your own puzzle is a challenging process! You'll have to do some tweaking if you want to create a good puzzle.

Notes

This challenge is worth 25 points because creating a puzzle is a pretty arduous task; you'll need to work hard in order to create one of each type. If you just need 20 points, you can do the three puzzles I have given, and create one puzzle of your own (either a "truth-tellers and liars" puzzle or a logic grid puzzle). If you just need 15 points, you can do the three given puzzles; you don't have to write your own.

Please write your solutions to Problems 1 through 3 as thoroughly as possible so that I can understand your solution process.

Challenge 9: Nerdy Jokes – 10 points

This is a relatively quick-and-easy challenge I devised to help your group out in case you need five or ten points to complete the project. Over the last few years I've been overjoyed to discover several comedians and cartoonists whose work relies (at least peripherally) on mathematics. In this challenge, you'll become acquainted with a couple of my favorites.

1. Check out xkcd.com and find a cartoon that requires some mathematical, scientific, or other technical knowledge on the reader's part in order to be funny. Research the technical terms and/or ideas used in the cartoon, and explain the cartoon as if to someone who doesn't understand the content. For this, please choose a cartoon that contains some ideas you didn't fully understand prior to working on this; I'm more interested in seeing you learn something than in hearing about what you already know.

2. Watch a show by stand-up comedian Demetri Martin (either his new Comedy Central show *Important Things* or his stand-up special, which also airs occasionally on Comedy Central). Occasionally he tells a set of jokes involving diagrams and graphs; look for a joke involving a graph (the kind you draw on coordinate axes, like the ones you dealt with in Section 2.2). Analyze this graph and discuss whether the graph makes sense mathematically. (Some of Demetri's graphs are quite sound mathematically; some aren't.)

Notes

Like a lot of comedy, both XKCD and Demetri Martin's acts involve bathroom and sexual humor. By undertaking this challenge, you absolve your friendly instructor of responsibility for any offensive material you might find while looking for nerd jokes. If you are sensitive to R-rated content or easily offended, please don't do this challenge.

Challenge ∞: Create Your Own Challenge – Point value to be determined

While I hope that your team can find several interesting challenges among the ones that I have provided, I also want to give you the freedom to explore a mathematical topic of your group's choosing if there is something specific you wish to explore.

If you want to try this, one approach your group can take is to write up a "challenge prospectus" in which you explain what your team would like to do, and how you will go about doing it. I will then either approve your prospectus, suggest some modifications, or reject it. (I will only reject a prospectus if I feel that the proposed challenge is too similar to already-existing ones, or if it cannot be modified into a workable challenge.) I will also propose a point value not exceeding 25 points for the challenge, based on the amount of work that I think will go into completing the challenge. If we (myself and your group) reach an agreement, then congratulations! You have created your own challenge.

Another possibility is for you to have someone in your group e-mail me and tell me about a topic or type of challenge you might enjoy, and I can try to come up with a suitable challenge (which I may then make available to everyone). However, given the volume of work I am dealing with at this time, I can't guarantee a quick turnaround.

Each team may do only one student-created challenge. You are not required to do a student-created challenge if you are happy with the ones I have provided.