

M316K – Foundations of Arithmetic
Spring 2009
Problem Set 1 – Due Friday, January 30

“Thought is only a flash between two long nights, but this flash is everything.” – Jules Henri Poincaré



You’ll notice that this problem set consists of about a dozen problems, and you have about a week and a half to do them. If you think back to your high school math classes, you’ll probably remember having a lot more problems to do, and a lot less time to do them. But don’t let this lull you into complacency! These problems aren’t as straightforward as the ones you probably had in most of your high school math classes – each problem requires some thought and exploration, and I want your solutions to be written neatly, thoroughly, and **in complete sentences** unless the situation obviously calls for a shorter answer.

Important note: You may be used to turning off the “common sense” center of your brain when working on math homework. That’s not your fault; it’s because you’ve probably spent most of your high school years working on math problems that don’t have any real-world value.¹ However, I plan to give plenty of problems with real-world value in this class, and on those problems, I hope and expect that you will bring your common sense to the table. Feel free to look for hidden assumptions or subtleties that need to be taken into account in these problems. I may even give you extra credit for pointing them out, assuming that you don’t use “common sense” to avoid doing the mathematics contained in the problems.

Section 1.4: 5, 6*, 12, 15, 16, 19, 22, 25, 28, 34, 36, 39*, 40.

On Problem 6, be sure to solve the problem that you have written. Try to write a problem about a topic other than the ones in Problems 1–5 – barnyard animals, wheels, and ticket sales.

I’d like for you to explore some things in Problem 39 that aren’t suggested by Bassarear. Instead of doing the four parts of the problem given in the book, please do the following:

1. First, do the part (b) given in the text.
2. In the table given in part (c), explain what the student did as if you were explaining it to a student in a class you were teaching (*i.e.* don’t assume straightaway that the reader “gets it”).
3. Describe several patterns that you see in the table.
4. Find all the possible amounts of (pretend) money that Lori and Betty could make with their (pretend) sale. Give these amounts in as nice and concise a way as you can; don’t simply list the entries in the table in the order in which they appear.
5. Assume that Lori and Betty have a limitless supply of 30-cent tags and 40-cent tags, and a limitless supply of (pretend) sale items to attach them to. Find all the possible amounts of (pretend) money that Lori and Betty can make in this situation. Prove your answer. (Don’t be intimidated by the word “prove”; all I’m asking for here is for an explanation that would convince a skeptical reader.)

Bonus Problems

B1. Fill in the array of squares on the attached page so that each row and each column of the array contains the numbers 1 through 6, and so that the numbers comply with the “less than” and “greater than” signs between adjacent cells. (This is a number puzzle that is often featured in Japanese Sudoku magazines; I couldn’t find any of this particular kind of puzzle on this side of the Pacific, so I made up my own. One number is already filled in. If you make a mistake and want to start over, ask me for another

¹Can you recall a situation in your life, other than a math class or math test, in which you had to solve the equation $x^2 - 3x - 10 = 0$? Probably not; and if you did, I’d be delighted to hear about it.

copy of the puzzle.) You don't need to write anything else on this problem; just give me the array of numbers.

- B2.** Suppose a particularly inquisitive student in a class you are teaching brings you a puzzle like the one given in Problem B1 and asks you for hints on how to solve such a puzzle. Explain a few techniques that might help your student get started. (In real life, I would hope that you would give hints sparingly at first, and supply more information only if it's clear that the student won't make any further progress on her own. For purposes of this exercise, we're assuming that the student really does need a few hints.)