

M316K – Foundations of Arithmetic
Spring 2009
Problem Set 3 – Due Friday, February 13

“The constructs of the mathematical mind are at the same time free and necessary. The individual mathematician feels free to define his notions and set up his axioms as he pleases. But the question is will he get his fellow mathematician interested in the constructs of his imagination. We cannot help the feeling that certain mathematical structures which have evolved through the combined efforts of the mathematical community bear the stamp of a necessity not affected by the accidents of their historical birth. Everybody who looks at the spectacle of modern algebra will be struck by this complementarity of freedom and necessity.” – Hermann Weyl



There are ten problems this time (not including bonus problems), spanning two sections. I think you’ll find some of these problems to be easier after you’ve read the corresponding sections. You’ll also find that some of the problems (such as Problem 13 in Section 2.1) are similar to other problems whose answers are given in the back of the book. You may want to use those as examples or practice problems if you’re having trouble. Of course, you should also feel free to ask questions of your friendly instructor!

The concepts covered in this problem set (sets and functions) are a bit beyond the scope of elementary mathematics, but I tried to choose problems that engage thinking skills that you’ll find useful. Venn diagrams are very useful pictorial representations that students can use in math as well as in other subjects. (I once had a teacher in elementary school who asked us to make a Venn diagram of the characters in *The Wizard of Oz*; the two regions in the diagram were for “good” characters and “evil” characters. The intersection of the two regions was for characters, such as the Wizard, who weren’t clearly good or evil. At least that’s how I remember it.) Thanks to standardized testing, students in your classes will be tested (over and over) on their ability to interpret graphs, including “line graphs” like the ones near the end of section 2.2.

Section 2.1: 13, 18, 20, 22.

Section 2.2: 11*, 14, 18, 22, 34, 35.

On part (e) of Problem 11, make assumptions that you consider reasonable in order to give a “ballpark” answer.

Bonus Problems

B1. Arnold has 27 twenty-pound weights in his weight room. He remembers that one of these weights is faulty; due to a manufacturing defect, it weighs a slight amount less than the other 26 weights. However, the defective weight has gotten mixed up with his other twenty-pound weights. The difference between the faulty weight and the other weights is so small that Arnold can’t detect it just by picking the weight up, or even by weighing it on a scale. However, Arnold is able to put the twenty-pound weights on a barbell (as many as he wishes on each side) and lift the barbell. Because he is an experienced weightlifter, he is always able to tell if one side of his barbell is heavier than the other, even if the difference is very small. There is a strategy Arnold can use to determine which weight is defective in only three lifts of his barbell. Find this strategy, and explain it as if you were explaining it to a certain governor whose brain is not his most well-developed muscle.¹

B2. Suppose you have sixteen square tiles. These tiles come in four different colors: red, blue, yellow, and green. Each tile has an image of an animal engraved on it: a cat, a dog, an eagle, or a fish. For each color-animal combination, there is exactly one tile bearing that combination; for example, there is exactly one red cat tile, exactly one yellow fish tile, and so on. Show that it is possible to arrange

¹Actually, I think the governor alluded to in this puzzle is an excellent leader with an exemplary grasp of economic and social policy. (There, hopefully that will lessen my chances of getting ~~terminated~~ beaten up.)

the tiles in a four-by-four grid so that each row of the grid contains all four colors *and* all four animals, and each column of the grid contains all four colors *and* all four animals.