

M316K – Foundations of Arithmetic
Spring 2009
Problem Set 4 – Due Friday, February 20

“Take from all things their number and all shall perish.” – Isidore of Seville



This week’s homework is very important because the topic, numeration, serves as the basis for everything that we will do in Chapter 3. In this section, you’ll explore several different numeration systems that were used before the Hindu-Arabic system became widespread. You’ll also learn a little bit about the idea of writing numbers in bases other than base 10, but unfortunately Bassarear’s account of this in the text is minimal; he reserves most of this for Exploration 2.9, which we will do on Wednesday.

On this problem set, many of the problems have partial answers listed in the back of the book. You are only required to write solutions to the parts whose answers are *not* given in the back of the book. I strongly recommend that you use the parts whose answers are listed to check that you understand how to do the problems before you start working on the required parts.

Section 2.3: 2, 3, 8, 12, 15, 16, 26, 27, 34, 37, 39.

On Problems 2 through 16, answers alone will suffice; you don’t need to explain your answers.

Bonus Problems

B1. It is the year 20X6, and three scientists on the planet NeoTerra need to transport themselves and three robots to the planet Zorblox-72C. Because of a long history of insufficient funding for science and engineering programs (which, legend has it, began sometime near the beginning of the millennium), the scientists are equipped only with a two-seat spaceship. Since the robots are human-sized and extremely fragile, they must ride in seats on the spaceship, just as the scientists do. There is one more potential problem: the robots have developed the ability to “think” about machines, which has made them self-aware, which in turn has made them capable of forming malicious intent against their creators.¹ Therefore, if at any time (no matter how brief) there are more robots than scientists on either planet, the robots will stage an uprising and kill the scientist(s). (Note that in this problem, the spaceship does not act as a “safe zone”; if a scientist and a robot land on a planet that is presently occupied by another robot and no scientists, he’s done for.) Devise a strategy that will allow the scientists to transport themselves and the robots safely to Zorblox-72C, assuming that the robots are able (and willing) to pilot the spaceship. For an extra point, devise a strategy that will work even if one of the robots is incapable of piloting the spaceship. For yet another extra point, devise a strategy that will work even if two of the robots are incapable of piloting the spaceship.

B2. When I was young, I enjoyed devising and using codes to communicate with my friends. The message below has been encoded using a *Caesar cipher*. Use the internet to find out what a Caesar cipher is (you’ll probably find that the Wikipedia page is informative enough) and decode the question. Answer the question, and encode your answer using the same Caesar cipher that I used. (Give your answer in encoded form *only*; after all, this is top-secret stuff.)

DOF KPK AOL JOPJRLU JYVZZ AOL HTBZLTLUA WHYR?

(*Hint:* After you learn what a Caesar cipher is, it will probably be very useful to think about what the first letter of the sentence should be.)

¹Don’t believe me? We’re not that far off. See the abstract for Professor J. Strother Moore’s talk at <http://www.ma.utexas.edu/cgi-pub/seminar/calendar?year=2009&month=02&day=11>.