M316K – Foundations of Arithmetic

Spring 2009

Problem Set 5 – Due Friday, February 27

"The different branches of Arithmetic – Ambition, Distraction, Uglification, and Derision." – Lewis Carroll, Alice in Wonderland

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In this problem set, we begin exploring the algorithms of grade-school arithmetic, beginning with addition. You'll actually learn several algorithms for addition, some of which allow you to add numbers mentally (without writing down your calculations).

On this problem set, some of the problems have partial answers listed in the back of the book. You are only required to write solutions to the parts whose answers are *not* given in the back of the book. I strongly recommend that you use the parts whose answers are listed to check that you understand how to do the problems before you start working on the required parts.

As with previous problem sets, there may be some problems in which there really isn't much to explain. However, if there is any doubt, you should always err on the side of explaining things more, not less. In particular, in the problems towards the end of the section in which you're figuring out the digits in a certain addition problem, I want you to pay special attention to explaining *how* you came up with the digits.

Section 3.1: 1*, 4, 7, 9, 10, 13, 17*, 19, 20*, 21, 24*.

In problem 1, I want you to explain *why* the pattern happens, not just explain what the pattern is or show a part of the table in which it occurs. (Bassarear's discussion of the pattern of the two-by-two squares is a good model of the kind of explanation I'm looking for.)

In problem 17, you don't have to find all of the possible ways to make 500; four different ways will suffice.

In problem 20, please find *all* possible values of a, b, and c. There is more than one way for the sum to work out.

In problem 24, just do parts (a) and (b).

Bonus Problems

B1. Arnold the weightlifter is back, and this time he has thirteen 50-pound weights, one of which is defective. Arnold remembers that the defective weight is a little bit heavier or a little bit lighter than the others, but he doesn't remember which. Help Arnold figure out how to find the defective weight in at most three lifts of his barbell. (If you worked on the first barbell puzzle in Problem Set 3, you should remember the rules: Arnold can't detect the defective weight just by picking it up and noticing that it seems a little too heavy or a little too light. He can only put weights on his barbell and determine whether one side of the barbell is heavier or lighter than the other.)

B2. Team Leibniz has taken Cody's advice for working in groups – have fun and get food! – and has decided to have a team homework/study session at Mangia Pizza[1] on Guadalupe. Brandin and Bonnie paid for the last two meals, so now it's up to Dakota or Erin to pay for the pizza. To decide who pays for the pizza, Dakota and Erin decide to play a game. They get ten sugar packets out of a container, and put the ten packets in a pile on the table. Starting with Dakota, the two take turns removing either one or two packets from the pile. Whoever removes the last sugar packet wins the game; the loser has to pay

[1]The next wave in creative teaching: advertisements embedded in problem sets! Cha-ching!

for the meal. On each turn, a player must remove either one or two packets; "passing" or skipping a turn is not allowed.

1. Assuming both players play as well as possible, which player will win? Tell Dakota or Erin what she needs to do in order to ensure her victory. (Be sure to explain what she should do at each stage of the game.)
2. What if we start with eleven sugar packets? What if we start with twelve? Thirteen? Determine which player has a winning strategy in each situation. (In each situation, Dakota is the first player to move.)
3. What if each player is allowed to remove one, two, or three sugar packets when it is her turn?