M316K – Foundations of Arithmetic

Spring 2009

Problem Set 9 – Due Friday, April 3

"Science is built up with facts, as a house is with stones. But a collection of facts is no more a science than a heap of stones is a house." – Jules Henri Poincaré

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This week, we'll be exploring divisibility, prime factorizations, and some other concepts in number theory. (You may not have heard the definition of the phrase "number theory" before; at its most basic level, number theory is the study of the integers and their properties.)

This week's problem set follows a different format than usual. I've written ten problems for you to work on; all of these problems are similar to exercises in Sections 4.1 and 4.2 (and some of them are taken directly from the book). I'm using this format for the problem set this week because most of the best problems in these sections have complete answers listed in the back of the book. I'd like for you to be able to use those for practice, but I also want to see if you can solve similar problems without having an answer key for guidance.

1. Do Problem 2 in Section 4.1. On (a), I'm looking for a proof of the caliber of Bassarear's proof that the sum of two odd numbers is even. That means you need to interpret the ideas algebraically and come up with a good, airtight argument. You don't have to prove your answer to part (b), but make sure to give a rule that predicts, for any set of integers, whether the product will be odd or even.

2. The following problems are based on Problem 17 in Section 4.1. In each problem, there may be more than one possible value of the missing digit(s). Be sure to show your work.

(a) If the three-digit number $5A7$ is divisible by 3, then what is the value of the digit A?
(b) If the four-digit number $734B$ is divisible by 9, then what is the value of the digit B?
(c) If the five-digit number $2C0C2$ is divisible by 9, then what is the value of the digit C?
(d) If the six-digit number $4D91E6$ is divisible by 72, then what are the values of D and E?

3. Do Problem 19 in Section 4.1. There is actually more than one possible answer, but I only need you to give one.

4. Do Problem 27 in Section 4.1. Two of the answers are given in the back of the book; you don't have to do those.

5. The following problems are based on Problem 32 in Section 4.1.

(a) What is the units digit of 7^{49}?
(b) What is the remainder when 7^{102} is divided by 2? **(c)** What is the remainder when 7^{155} is divided by 5? **(d)** What are the tens and units digits of 7^{2009}?

6. Find the prime factorization of each of the following numbers: 24, 50, 99, 105, 2000, 16632. For some examples with answers given in the back of the book, see Problem 3 in Section 4.2.

7. For each of the following numbers, determine whether the number is prime or composite. Please follow the instructions in Problem 4 of Section 4.2, and explain what you're doing. The numbers are: 73, 85, 91, 141, 233, 817, 2009.

8. Do Problem 10 in Section 4.2. By "construct," Bassarear means "give an example of."

9. Do Problem 13 in Section 4.2.

10. Do Problem 17 in Section 4.2. As before, don't be intimidated by the word "prove"; I'm just looking for a good mathematical explanation that would convince a skeptical listener.

Bonus Problems

B1. Solve Problem 13 in Section 4.1. Remember, in order to get credit, you need to explain your reasoning carefully! (*Hint:* The key to figuring out this problem is to assume that Bernice is very good at logic (and knows Alfie's street address), and realize that Bernice's ability or inability to solve the problem at a given time is an important piece of information for you to use. I call puzzles of this type "metapuzzles" because they are puzzles about a fictional character's ability to solve a puzzle. However, the word "metapuzzle" has taken on several different meanings.)

B2. Solve Problem 18 in Section 4.2.