M316K – Foundations of Arithmetic

Spring 2009

Problem Set 11 – Due Monday, April 20

"We are not very pleased when we are forced to accept a mathematical truth by virtue of a complicated chain of formal conclusions and computations, which we traverse blindly, link by link, feeling our way by touch. We want first an overview of the aim and of the road; we want to understand the idea of the proof, the deeper context." – Hermann Weyl

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This problem set is a bit truncated, because I know you will want to spend the weekend studying for the exam. However, the content is important: we're beginning our work on fractions, which is a topic that I think is usually not taught as carefully and as thoughtfully as it should be.

Section 5.2: 1*, 4, 6*, 7, 8, (11), 28, 30*.

In Problem 1, you don't have to do the parts in the back of the book. On the other six, please note the instruction at the beginning of the problem to "justify your answer." This is important.

On Problem 6, the answer is not $\frac{1}{5}$. So answer the following question (in addition to the one Bassarear asks): if you asked a student in your class what fraction the shaded region represents, and he/she said "one fifth," what would you tell him/her? (I hope you wouldn't simply say "Wrong!" and leave it at that.)

You don't have to turn in Problem 11, but make sure you can do these fairly efficiently, and by hand (not using a calculator).

On part (a) of Problem 30, I'll give you bonus points if you solve the problem without using algebra (*i.e.*, without setting up an equation or using variables), as long as you explain your method. On (b), Bassarear is asking you to estimate the fraction of students who are in the chorus with a unit fraction, because the exact answer is not a unit fraction. (A unit fraction is a fraction where the numerator is 1.)

Bonus Problems

B1. In the attached puzzle, fill each square with one of the numbers from 1 to 8 so that:

- Each row and each column contains all eight numbers from 1 to 8.
- Each block of squares (marked by the bold lines) has the correct value under the correct operation. For example, the four numbers in the L-shaped region in the upper left have to add up to 19. The five numbers in the L-shaped region in the lower right have to multiply to 1792. The "2÷" on the left means that when you divide the two numbers in that box, the quotient should be exactly 2, but the smaller number can go on top or on the bottom. Similarly, in a subtraction region, the two numbers can be in either order (smallest first or largest first).

You **are** allowed to have the same number twice within a region, as long as it doesn't appear twice in a row or twice in a column. Even if you aren't able to finish the puzzle, I will give partial credit for partial progress.

B2. Tell me what ideas you used to fill in the squares you figured out in the puzzle in problem B1. The better the explanation, the bigger the bonus.