M305G – Precalculus

Summer II 2008

Problem Set 3 – Due Monday, July 21

"The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colors or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics." – Godfrey H. Hardy

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Another helpful hint on the homework:

If you find yourself getting stuck on an even-numbered problem in the textbook, you may find it useful to try doing the odd-numbered problem before it. The book contains the answers to most (if not all) of the odd-numbered problems. Even though the answer key doesn't explain how one arrives at the correct answer, you might be able to get a sense of what's going on by looking at the answer . . . and that may help you figure out what to do on the even-numbered problem I assigned.

However, **be careful** about how you use the answers. The best way to use the answers (if you use them at all) is to look at an answer after you've made a serious attempt to work the problem without it. Then, if you get stuck, you might take a look at the answer and see what it is you're trying to get to. Then – and this is very important – seek to understand *why* the answer is what it is. Don't just look at the answer, try to observe a relationship with the information given in the problem, and use that pattern to "guess" what the answer to the next problem should be. One of the greatest false complaints about mathematics is that it's full of exceptions, places where an unscrupulous textbook (or teacher) can throw a curveball and try to trip you up. It's not. Ninety-nine percent of all perceived "exceptions" in mathematics are a result of not fully understanding the underlying mathematics.[1] So in other words, if you think you have a right answer but still aren't totally sure *why* it is the right answer, come and talk to me so I can give you the full story about what's really going on.

That's the pep talk for today. Here is the problem set. This covers the stuff we'll do in class on Thursday.

Section 2.4: 28, 34, 36, 38, 42, 44, 50, 56.

Section 2.5: 28, 30, 36, 41, 45, 51, 56, 60, 62, 70, 74.
I assigned some odd-numbered problems here so that you could start off the graph-transformation section with a few problems where you can check what you're doing. Keep in mind that there are steps that you'll need to show that aren't in the back of the book!

Also: Since this weekend's homework is relatively short, I have an additional task for you. There's nothing to turn in here, but this is still very important. Take a look at the appendix, sections A.6, A.9, and A.5 (listed roughly in order of priority, with A.6 being most important right now). Make sure you are familiar with the techniques discussed here; the best way to test yourself is to try some of the odd-numbered exercises and make sure you're doing things correctly. These are all things we are likely to use next week, and I probably won't have time to give them special attention in class. This is especially important if you have gone a while without taking a math class. If you look at all of this and feel completely overwhelmed, you need to see me ASAP.

Bonus Problems

B1. One of the biggest, nastiest examples of a piecewise-defined function that you'll ever find is in the United States tax code. Download the Form 1040 Instructions from www.irs.gov and take a look at the tax tables on pages 63 through 74. The first of the four columns in each table is for single taxpayers; we'll focus on that. These twelve pages make up the definition of a function $T(x)$, the amount of tax

[1] Also, ninety-nine percent of all statistics in everyday conversation are made up, but that's another story.

owed by a single taxpayer whose taxable income is x dollars. Consider the function $N(x) = x - T(x)$. What real-world significance does the function $N(x)$ have? (That is, explain what the function $N(x)$ "means.") Find two numbers x and y such that $x < y$, but $N(x) > N(y)$. Explain why this is strange, and perhaps a little infuriating. (True story: Your friendly instructor was a victim of this phenomenon this spring!)

B2. Consider the graph of the function $f(x) = \sqrt[3]{x}$. Find the function $g(x)$ whose graph is obtained from the graph of $f(x)$ by shifting it to the right 3 units, then stretching the resulting graph horizontally by a factor of 2 about the y-axis. Then, find the function $h(x)$ whose graph is obtained from the graph of $f(x)$ by stretching it horizontally by a factor of 2 about the y-axis, then shifting the resulting graph to the right 3 units (that is, performing the same two transformations in the reverse order). Are $g(x)$ and $h(x)$ the same function? (Note that I want algebraic definitions for $g(x)$ and $h(x)$ here – not just graphs.)

B3. Consider the piecewise-defined function

$$f(x) = \begin{cases} 3 & \text{if } x \leq 1 \\ x+2 & \text{if } x > 1 \end{cases}$$

Find a definition of $f(x)$ that expresses $f(x)$ in terms of x using a single formula (*i.e.*, not a piecewise definition). (*Hint:* You'll need to use an absolute value in your formula. When deciding what to put in the absolute value, it is helpful to graph the function $f(x)$, and notice where the "turning point" is.)