

M305G – Precalculus  
Summer II 2008  
Problem Set 5 – Due Friday, July 25

*“We arrive at truth, not by reason only, but also by the heart.” – Blaise Pascal*



This problem set will be relatively short, since it only covers the material over polynomials from Tuesday. I'm including several bonus problems so that you can make up some of the points you missed on the previous problem set.

**Section 4.1:** 33, 34, 38, 43, 68, 76, 82, 86.

Problems where you have to graph polynomials will be valued higher than 2 points, since there are several things to do on each such problem. Be sure to show your work and follow the methods we used in class to analyze end behavior, number of intercepts, number of turning points, and behavior near  $x$ -intercepts (multiplicities and the “approximation” method).

**Bonus Problems**

**B1.** Suppose that  $f(x)$  is a fifth-degree polynomial function. Explain why the graph of  $f(x)$  cannot go through all six of the following points:  $(1, 1)$ ,  $(3, 1)$ ,  $(5, 1)$ ,  $(7, 1)$ ,  $(9, 1)$ , and  $(11, 1)$ .

**B2.** There are several cool theorems about polynomials that we didn't cover in class since they're beyond the scope of this course: Descartes' Rule of Signs, the Rational Root Theorem, and Eisenstein's Criterion. Look these theorems up (the Wiki articles should be adequate), and use the three theorems (not necessarily in the order given) to prove the following facts:

1. The polynomial  $f(x) = x^4 + 8x^3 - 5x + 1$  does not have any rational roots.
2. The polynomial  $g(x) = x^4 + 9x^3 - 6x^2 + 18x - 39$  does not have any rational roots.
3. The polynomial  $h(x) = x^5 + 3x^3 - 2x^2 + 5x - 10$  does not have any negative real roots.

**B3.** Perform the following polynomial multiplications:  $(1 - x)(1 + x + x^2)$ ,  $(1 - x)(1 + x + x^2 + x^3)$ , and  $(1 - x)(1 + x + x^2 + \cdots + x^{10})$ . Use your findings to calculate the sum of the following series:

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{1024}$$

(Note that  $1024 = 2^{10}$ .)

**B4.** The graph of a certain fourth-degree polynomial function  $f(x)$  passes through the points  $(-3, 0)$ ,  $(-1, 0)$ ,  $(0, 4)$ ,  $(1, 0)$ , and  $(3, 0)$ . Give a formula for  $f(x)$ . (It is worth noting that there is only one fourth-degree polynomial function whose graph passes through these points.)