

Emerging Scholars Program – Fall 2007
M210E – Calculus Workshop
Problem Set 10

“Logic is the hygiene the mathematician practices to keep his ideas healthy and strong.” – Hermann Weyl



55. Close enough for government work. Write the Taylor series for the function e^x , centered at $x = 0$. Suppose we want to use a Taylor approximation to compute the value of $1/e$, correct to six decimal places. How many terms of the Taylor series would we need to use to do this? What if we wanted to compute the value of e , correct to six decimal places?

56. No need for a calculator. Use Taylor approximations to compute the following values by hand, each correct to three decimal places.

$$\sqrt[3]{28} \quad \sin 0.1 \quad \log 1.2 \quad \tan^{-1} 20$$

57. It won't integrate, but ... Compute the value of the following integral, correct to two decimal places.

$$\int_0^1 \frac{\cos x}{1+x^2} dx$$

58. Vector geometry: A sneak preview. Geometrically speaking, a *vector* in the coordinate plane is a directed line segment (*i.e.* an arrow) that runs from one point in the plane to another. Vectors are typically drawn from the origin unless otherwise specified; however, a vector by itself does not have a well-defined starting point or endpoint. A vector, like a point, is described by a pair of coordinates, but it is generally good style to use a different notation for vector coordinates than we do for the coordinates of a point. (*Author's note:* I tend to use angle brackets for the coordinates of a vector.) For example, the vector $\langle 3, -4 \rangle$ is usually drawn from the point $(0, 0)$ to the point $(3, -4)$, but it could also be drawn from the point $(2, 5)$ to the point $(5, 1)$. Since the direction of a vector is the same no matter how one draws it, it makes sense to talk about two vectors being perpendicular. Suppose we are given two vectors $\langle x_1, y_1 \rangle$ and $\langle x_2, y_2 \rangle$. Give – and prove – a condition on the coordinates x_1, y_1, x_2 , and y_2 that can be used to determine whether these two vectors are perpendicular.

59. An acute observation. See if you can refine the theorem you proved in the previous problem to give a condition that can be used to determine whether two vectors meet at an acute angle, a right angle, an obtuse angle, or a straight angle; or if they run in the same direction.

60. Ingenuity: Cause for alarm? Recently, a rare but potentially deadly disease called *calculitis* has begun to afflict freshmen at colleges and universities across the United States. Fortunately, the disease can be treated if it is detected early. One way to determine whether a freshman has calculitis is to shave the patient's head and apply a chemical to his/her roots, which is supposed to cause the hair roots to turn purple if the patient is infected. This test is called the *root test*. The root test has a 98% chance of returning a correct result if the patient is not infected, and a 99.9% chance of returning a correct result if the patient is infected. Approximately 0.2% of college students become infected with calculitis at some point in their first year of college. If a college freshman with no prior symptoms of calculitis takes the root test and receives a positive result, what is the probability that he/she is actually infected?