Emerging Scholars Program – Fall 2007
M210E – Calculus Workshop
Problem Set 11

"The knowledge of which geometry aims is the knowledge of the eternal." – Plato

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61. The almighty dot product. Use dot products to answer the following questions:

1. Is the angle between the vectors $\langle 2, 1, -3\rangle$ and $\langle 1, 4, 0\rangle$ acute, right, or obtuse?
2. Let $A = (1, 1, 4)$, $B = (0, -1, -1)$, and $C = (2, -7, 1)$. Is the angle $\angle ABC$ acute, right, or obtuse?
3. Suppose that $\mathbf{v}$ and $\mathbf{w}$ are vectors with $|\mathbf{v}| = |\mathbf{w}| = 4$ and $\mathbf{v} \cdot \mathbf{w} = 8$. What is the angle between $\mathbf{v}$ and $\mathbf{w}$?
4. Let $A = (1, 3, 0)$, $B = (-1, 0, -2)$, and $C = (4, 0, -1)$. If these three points are joined by line segments to form the triangle ΔABC, what are the angles of this triangle? (You'll probably need to use a calculator to get good approximations here.) Verify that the three angles add up to $180°$.
5. Find the scalar projection and the vector projection of $\langle 2, -5, 1\rangle$ onto the vector $\langle -3, -3, 2\rangle$.
6. Find the scalar projection and the vector projection of the vector $\langle x, y, z\rangle$ onto each of the three standard basis vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.
7. Find all vectors in the yz-plane that are perpendicular to the vector $\langle 1, 2, 3\rangle$.

62. Plane-ly perpendicular. You may recall from your prior mathematical experience that a plane in coordinate space is given by an equation of the form $ax + by + cz = d$, where a, b, c, and d are real-number constants. Find an equation, in this form, for the plane through the origin that is perpendicular to the vector $\langle 2, 1, -1\rangle$. Then find an equation, in this form, for the plane through the point $(3, 4, 5)$ that is perpendicular to the line passing through the points $(-1, 0, 1)$ and $(2, 2, 4)$.

63. Angles between planes. Consider the planes given by the equations $x + 3z = 0$ and $2x - y - z = 0$. For each of these planes, give a vector that is perpendicular to the plane. Then use these vectors to determine the acute angle at which the two planes meet. (Again, you'll probably need to use a calculator if you want an answer in degrees.)

64. Geometry meets chemistry.[1] A molecule of methane, or CH_4, consists of a carbon atom surrounded by four hydrogen atoms. The four hydrogen atoms (which in a methane molecule become positively-charged ions) are arranged so that they are as far from one another as possible, while remaining a fixed distance away from the carbon atom. The sharing of electrons between the carbon atom and the four hydrogen atoms creates four C–H bonds. Find the angle between two of these bonds.

65. A problem worth reflecting on. Suppose you are given a plane $\mathcal{P}$ passing through the origin in $\mathbb{R}^3$, and a vector $\mathbf{x}$. Let $\mathbf{r}$ be a vector perpendicular to $\mathcal{P}$. Find a formula, in terms of $\mathbf{x}$ and $\mathbf{r}$, that gives the projection of the vector $\mathbf{x}$ onto the plane $\mathcal{P}$. Then find a formula, in terms of $\mathbf{x}$ and $\mathbf{r}$, that gives the reflection of the vector $\mathbf{x}$ in the plane $\mathcal{P}$.

66. Ingenuity: Points of trisection. Let ΔABC be an equilateral triangle with sides of length 1. Let points D, E, and F lie on sides AB, BC, and CA, respectively, so that $AD = 2DB$, $BE = 2EC$, and $CF = 2FA$. Suppose we draw the line segments AE, BF, and CD. These line segments form a smaller equilateral triangle in the interior of ΔABC. What is the length of a side of this triangle?

[1] The author's last experience with chemistry was in high school, in a year that began with the number 1. Therefore, what you read here should not be taken as fact unless you verify it using a more authoritative source.