

Emerging Scholars Program – Fall 2007
M210E – Calculus Workshop
Problem Set 17

“In the fall of 1972 President Nixon announced that the rate of increase of inflation was decreasing. This was the first time a sitting president used the third derivative to advance his case for reelection.” – Hugo Rossi



- 97. I’m partial to partials.** Find the first and second partial derivatives of the following functions. (Some of the second partials may get rather ugly; if you feel like you’ve had enough practice, feel free to skip them. However, keep in mind that you can use the mixed second partials f_{xy} and f_{yx} to check that you have done the work correctly, since Clairaut’s Theorem states that f_{xy} should be equal to f_{yx} .)

$$(x^2 + 2xy + y)^3 \quad e^{xy^2} \quad \cos x \cos y + \sin x \sin y \quad \frac{x^2 + y^2 + 1}{xy - 1}$$

- 98. Anti-corporate propaganda.** The giant, greedy, soulless mega-corporation McRonald’s¹ wants to decrease the size of its ice cream cones so that the volume of ice cream needed to fill the cone to the brim is as small as possible. Since consumers *see* the linear dimensions of the cone (base diameter plus height) and not the volume, McRonald’s is willing to either decrease the base diameter by a small number ϵ , or decrease the height of the cone by the same small number ϵ . If the current cone has height 4 inches and diameter 1.5 inches, which change is more profitable for McRonald’s?

- 99. A ripple effect.** Sketch the surface described by the equation $z = \sin(x^2 + y^2)$, and find the equation of the tangent plane to this surface at the point $(\sqrt{\pi}, \sqrt{\pi}, 0)$.

- 100. Approximation in two variables.** Use linear approximation to estimate the value of the expression

$$e^{0.2 + \sin 0.3}.$$

Use a calculator to find out how close your approximation is to the real McCoy.

- 101. Ingenuity: Does it pay to be a flip-flopper?** Suppose you are on a game show where you are given a chance to win a new car. There are three doors; behind one door is a car, and behind the other two doors are steaming piles of – you guessed it – epsilons and deltas. You know the rules of the game in advance: you pick one door, but do not open it. The game show host (let’s say quintessential game show host Tom Bergeron, for lack of someone really interesting) then opens one of the other two doors, one which does *not* contain the car (he knows, of course, which door hides the car). You then have the choice of opening the door you initially chose, or switching to the other unopened door and opening it. You win whatever is behind the door you open. Is it better to open your original door, better to switch to the other door; or are the two strategies equally likely to pay off?

- 102. Ingenuity: Upgrading the cube.** What would a four-dimensional “cube” (often called a *hyper-cube* or *tesseract*) look like? How many vertices would it have? How many edges? Two-dimensional faces? Three-dimensional facets? What other three-dimensional figures have natural four-dimensional “cousins”?

¹No relation to an actual corporation, or to actual political views of your friendly AI.