Emerging Scholars Program – Fall 2007
M210E – Calculus Workshop
Problem Set 18

"In the fall of 1972 President Nixon announced that the rate of increase of inflation was decreasing. This was the first time a sitting president used the third derivative to advance his case for reelection." – Hugo Rossi

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103. Chain gang. Suppose that f is a function of two variables u and v, each of which is, in turn, a function of two independent variables x and y. Suppose we are given the following information:

$$u_x(0,0) = 3 \qquad u_y(0,0) = -1 \qquad v_x(0,0) = 2 \qquad v_y(0,0) = 10$$

If $\frac{\partial f}{\partial x} = 24$ and $\frac{\partial f}{\partial y} = -16$ at the point $(x, y) = (0, 0)$, what are the values of $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$ at the point $(u, v) = (u(0,0), v(0,0))$?

104. Think inside the box. Suppose we have a rectangular box with length 6 centimeters, width 4 centimeters, and height 3 centimeters. If we begin increasing the length of the box at a rate of 0.1 centimeters per second, increasing the width at a rate of 0.2 centimeters per second, and decreasing the height at a rate of 0.1 centimeters per second, at what rate is the length of the inner diagonal of the box increasing or decreasing when we begin this process?

105. We've got gas. The *ideal gas law* states that, for an ideal gas in an enclosure of finite volume,

$$pV = nRT,$$

where p is the pressure exerted by the gas on the walls of the container, V is the volume of the gas, n is the number of moles of gas in the container, T is the temperature of the gas in Kelvins, and R is the ideal gas constant, which is approximately $8.31\ \mathrm{m^3 \cdot Pa \cdot K^{-1} \cdot mol^{-1}}$. Suppose that we have ten moles of an ideal gas in a spherical balloon of radius 10 centimeters, kept at a temperature of 300 Kelvins. We then begin compressing the balloon and heating its contents simultaneously; the balloon's radius decreases at a rate of one millimeter per second, and the temperature increases at a rate of one Kelvin per second. At the moment when the balloon's radius is 9 centimeters, at what rate (in Pascals per second) is the pressure of the gas increasing?

106. Implicit differentiation goes 3D. Find the equation of the tangent plane to the surface defined by the equation $xy + xz + yz^2 = 1$ at the point $(1, 1, 0)$.

107. Directional derivatives: A sneak preview. If the vector $\langle 3, -4, c\rangle$ is tangent to the surface $z = x^2 - y^2$ at the point $(-2, 3, -5)$, what is the value of c?

108. Ingenuity: A Fibonacci decimal. Define

$$S = \frac{1}{10^1} + \frac{1}{10^2} + \frac{2}{10^3} + \frac{3}{10^4} + \frac{5}{10^5} + \frac{8}{10^6} + \cdots,$$

where the denominators of the fractions are the powers of ten, and the numerators are the terms of the Fibonacci sequence. What does S look like when written as a decimal? Is this decimal repeating or non-repeating?