

Emerging Scholars Program – Fall 2007
M210E – Calculus Workshop
Problem Set 21

“Attaching significance to invariants is an effort to recognize what, because of its form or colour or meaning or otherwise, is important or significant in what is only trivial or ephemeral. A simple instance of failing in this is provided by the poll-man at Cambridge, who learned perfectly how to factorize $a^2 - b^2$ but was floored because the examiner unkindly asked for the factors of $p^2 - q^2$.” – H.W. Turnbull



121. Integrals again? Evaluate the following double integrals:

$$\int_0^1 \int_0^1 x^2 y^3 \, dy \, dx \quad \int_0^{\pi/2} \int_{-1}^1 xy \cos(xy) \, dy \, dx \quad \int_{-1}^1 \int_{-2}^3 x^2 \sin y^3 \, dx \, dy$$

122. Integrals again, again? Evaluate the following double integrals:

$$\int_1^3 \int_y^{y^2} xy \, dx \, dy \quad \int_0^{\pi} \int_x^{2x} y \sin x^3 \, dy \, dx$$

123. Easier than you think. Evaluate the double integral

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1-x^2-y^2} \, dy \, dx.$$

124. Doubly improper. Evaluate the double integral

$$\int_1^{\infty} \int_1^y \frac{1}{x^4 + x^2 y^2} \, dx \, dy.$$

125. The Yellow Dart. Minnie “The Yellow Dart” Tao throws darts at the region in the coordinate plane bounded by the positive x -axis, the positive y -axis, and the parabola given by the equation $y = 2 - x^2$.

1. If Minnie is guaranteed to hit the region described on each throw, and each point in the region has an equal probability of being hit, what is the probability that a given dart will land above the line $y = 1$?
2. If Minnie is guaranteed to hit the region described on each throw, and each point in the region has a probability of being hit proportional to its distance from the y -axis, what is the probability that a given dart will land above the line $y = 1$?

126. Ingenuity: Who’s the better buyer? Cody’s Pizzeria, another new pizzeria on the drag, is unique in that it sells pizza by the pound, and allows customers to buy any amount of pizza they wish (even if it’s a non-integer or very small number of pounds).¹ The management also pays close attention to the prices of the ingredients and changes the price per pound of its pizza frequently, causing the price to increase and decrease periodically. Naureen and William are deputized by their respective ESP classes to buy pizza for the class each day. However, Naureen and William have different approaches to buying pizza. Naureen buys ten pounds of pizza each day, while William buys thirty dollars’ worth of pizza each day. On average, which buyer gets a better deal on the pizza?

¹Your friendly AI didn’t come up with this problem on his own; the original version of this problem involves flour rather than pizza. Could your AI simply quote the original problem with something like flour that makes sense being sold by the pound? Noooooo! He has to make it about something besides flour, because flour is too *boring*. (eye roll)