

Emerging Scholars Program – Fall 2007  
M210E – Calculus Workshop  
Problem Set 22

*“A formal manipulator in mathematics often experiences the discomfoting feeling that his pencil surpasses him in intelligence.” – Howard W. Eves*



- 127. Change-up.** Assume that  $f(x, y)$  is a continuous function on  $\mathbb{R}^2$ . Change the order of integration of each of the following integrals; that is, write the integrals in the reverse order, making the necessary changes in the limits of integration.

$$\int_0^1 \int_0^x f(x, y) \, dy \, dx \quad \int_0^1 \int_{y^2}^y f(x, y) \, dx \, dy \quad \int_{-2}^2 \int_{x^2}^4 f(x, y) \, dy \, dx \quad \int_0^{\pi/2} \int_0^{\tan x} f(x, y) \, dy \, dx$$

- 128. Know your limits.** Let  $R$  be the bounded region in  $\mathbb{R}^2$  between the parabola  $y = 4 - x^2$  and the line  $x - y + 2 = 0$ . Compute the double integral

$$\iint_R 2xy \, dA.$$

- 129. The Polar Express.** Express each of the following objects in terms of the polar coordinates  $r$  and  $\theta$ .

1. The coordinates  $x$  and  $y$ .
2. The circle  $x^2 + y^2 = 4$ .
3. The ratio  $\frac{y}{x}$ .
4. The line  $x = 2$ .
5. The hyperbola  $x^2 - 4y^2 = 1$ .

- 130. My hump.** Compute the double integral

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{x^2 + y^2 + 1} \, dx \, dy.$$

- 131. Some suggestive limits of integration.** Compute the double integral

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2 y^2 \, dy \, dx.$$

- 132. Ingenuity: The wrong hands at the right time.** Due to a mistake in the manufacturing process, an analog clock was made so that the hour hand and minute hand that both point to twelve at midnight, but turn at the wrong speeds. The hour hand turns at the speed that the minute hand is supposed to turn, and the minute hand turns at the speed that the hour hand is supposed to turn. At how many times during a twelve-hour interval does the clock display a time that “makes sense” (in terms of the placement of the hour hand and minute hand), but is not the correct time?