

Emerging Scholars Program – Fall 2007
M210E – Calculus Workshop
Problem Set 23

“The real end of science is the honor of the human mind.” – Carl Jacobi



133. Triple trouble. Evaluate the following integrals:

$$\int_1^e \int_1^e \int_1^e \log(xyz) \, dx \, dy \, dz \qquad \int_0^2 \int_x^{2-x} \int_0^{x+y} (xy + yz) \, dz \, dy \, dx$$

134. While we’re on the subject . . . Sketch the region of space over which the second integral above is evaluated, and find its volume.

135. Search for the centroid. A trapezoid in the coordinate plane has vertices at the points $(-a, 0)$, $(a, 0)$, $(b, 1)$, and $(-b, 1)$, for some positive real numbers a and b . Find, in terms of a and b , the centroid of the trapezoid. (The *centroid* of a planar object is the point that would be the center of mass of the object if the object had uniformly distributed mass.)

136. Moments of inertia: the disc. Suppose a thin disc has radius R , height h , and mass M . What is the moment of inertia of the disc about an axis through its center (perpendicular to the circular base of the disc)?

137. Moments of inertia: the sphere. Suppose a sphere has radius R and mass M . What is the moment of inertia of the sphere about an axis through its center?

138. Ingenuity: Really, this could be so much worse. Suppose that x , y , and z satisfy the following system of equations:

$$\begin{aligned} 23x + 42y + 15z &= 26 \\ 42x + 15y + 23z &= -18 \\ 15x + 23y + 42z &= 11 \end{aligned}$$

What is the value of $x + y + z$?