Problem Set # 12

M382E: Algebraic Topology

Due: November 25, 2008

Work backwards through the relative cup product in the last displayed equation on p.240 of Hatcher. Your journey backwards should take you to the bottom of p.209, to p.149, and perhaps beyond.

Problems in Hatcher

Section 3.3 (page 257): 9, 22, 25, 27

Other Problems

1. For each of the following manifolds write a basis for the integral homology modulo torsion and compute the intersection products.

   (a) S^4

   (b) $S^n \times S^n$, $n = 1, 2, 3, \ldots$

   (c) $CP^3 \times \mathbb{RP}^3$

   (d) $CP^3 \times \mathbb{RP}^2$

   (e) $(S^2 \times S^2)\#(S^2 \times S^2)$ (connected sum)

2. (a) Suppose W is a compact $(n+1)$-dimensional oriented topological manfold with boundary M, and $F\colon W \to X$ is a continuous map to a topological space X. Let $f\colon M \to X$ be the restriction of F to M. Prove that $f_*[M] = 0$ in $H_n(X)$. (You may want to read pp. 252–4 in Hatcher.)

   (b) For a smooth manifold M there are two constructions of an orientation double cover $\hat{M} \to M$: via frames of the tangent bundle (see Problem Set #1) and via the construction in Hatcher. Prove that there is a homeomorphism between the two covers which induces the identity map on M.

   (c) For any space X construct homomorphisms $MO_n(X) \to H_n(X; \mathbb{Z}/2\mathbb{Z})$ and $MSO_n(X) \to H_n(X; \mathbb{Z})$.

3. Use the theorem proved in class about intersections and cup products to solve problem #8 on page 258 in Hatcher. (You did that problem last week.)