Problem Set # 9

M382E: Algebraic Topology

Due: November 4, 2008

Read through the examples on cup products in Hatcher. We will move quickly to duality and then come back and compute examples more easily. But to work the exercises you will need to use the cup product structure on the cohomology of projective spaces.

Problems in Hatcher

Section 3.2 (page 228): 1, 2, 3, 6, 7, 8, 11, 14, 15

Other Problems

1. Let A, F, G be abelian groups.
   (a) Prove that $\mathrm{Ext}(F, A) = 0$ if F is free.
   (b) Prove that $\mathrm{Ext}(G, A) = \mathrm{Ext}(\mathrm{Tors}\, G, A)$ if G is finitely generated, where $\mathrm{Tors}\, G$ is the torsion subgroup of G.
   (c) Prove that $\mathbb{Q}$ is torsionfree but not free.

2. Find a Δ-set whose geometric realization is $\mathbb{RP}^2$ and use it to compute the cohomology rings $H^\bullet(\mathbb{RP}^2; \mathbb{Z})$, $H^\bullet(\mathbb{RP}^2; \mathbb{Z}/2\mathbb{Z})$, and $H^\bullet(\mathbb{RP}^2; \mathbb{Z}/4\mathbb{Z})$.

3. Complete the proof that if Γ is a discrete abelian group and X the geometric realization of a Δ-set, then $H^1(X; \Gamma)$ is isomorphic to the set of principal Γ-bundles over X up to isomorphism. Can you extend to a general CW complex X?