1. Here are some problems concerning invertibility in symmetric monoidal categories, as in Lecture 17.
   (a) Construct a category of invertibility data (Definition 17.18), and prove that this category is a contractible groupoid.
   (b) Prove Lemma 17.21(i).
   (c) Let $\alpha: \text{Bord}^{SO}_{(0,1)} \to C$ be a TQFT. Prove that if $\alpha(\text{pt}_+) \in C$ is invertible, then $\alpha$ is invertible.

2. Compute the invariants of the Picard groupoid of superlines. (See (17.27) and (17.35) in the notes.)

3. Show that a special $\Gamma$-set determines a commutative monoid. More strongly, construct a category of special $\Gamma$-sets, a category of commutative monoids, and an equivalence of these categories.

4. Let $S$ denote the $\Gamma$-set $S(S) = \Gamma^{\text{op}}(S^0, S)$, for $S \in \Gamma^{\text{op}}$ a finite pointed set. Compute $\pi_1[S]$.

5. Let $C$ be a category. An object $* \in C$ is initial if for every $y \in C$ there exists a unique morphism $* \to y$, and it is terminal if for every $y \in C$ there exists a unique morphism $y \to *$.
   (a) Prove that an initial object is unique up to unique isomorphism, and similarly for a terminal object.
   (b) Examine the existence of initial and terminal objects for the following categories: Vect, Set, Space, Set$_+$, Space$_+$, the category of commutative monoids, a bordism category, a category of topological quantum field theories.
   (c) Prove that if $C$ has either an initial or final object, then its classifying space is contractible.

6. Let $K$ denote the classifying spectrum of the category whose objects are finite dimensional complex vector spaces and whose morphisms are isomorphisms of vector spaces. Compute $\pi_0K$. Compute $\pi_1K$.

7. Let $M$ be a commutative monoid. We described a general construction of the group completion of any monoid. Give a much simpler construction of the group completion $|M|$ by imposing an equivalence relation on $M \times M$. You may wish to think about the examples $M = (\mathbb{Z}^\geq 0, +)$ and $M = (\mathbb{Z}^>, \times)$. 
8. Let $G$ be a topological group, viewed as a category $C$ with a single object. (Normally we use ‘$G$’ in place of ‘$C$’, but for clarity here we distinguish.)

(a) Describe the nerve $NC$ of $G$ explicitly.

(b) Define a groupoid $\mathcal{G}$ whose set of objects is $G$ and with a unique morphism between any two objects. Construct a free right action of $G$ on $\mathcal{G}$ with quotient $C$. First, define carefully what that means.

(c) Prove that the classifying space $B\mathcal{G}$ is contractible.

(d) Show that $G$ acts freely on $B\mathcal{G}$ with quotient $BC$.

So we would like to assert that $B\mathcal{G} \to BC$ is a principal $G$-bundle, and by Theorem 6.45 in the notes a universal bundle, which then makes $BC$ a classifying space in the sense of Lecture 6. The only issue is local triviality; see Segal’s paper.