Differentiable structures on metric measure spaces

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Abstract: Rademacher's theorem on the almost everywhere differentiability of real-valued Lipschitz functions on $\mathbb{R}^n$ can be generalized to metric measure spaces for which the measure is doubling and such that between every pair of points there exist "sufficiently many" curves of finite length. Examples include fractals such as nilpotent Lie groups with Carnot metrics. It follows that there is a unique bi-Lipschitz invariant differentiable structure, which enables one to do first order calculus. This has applications, e.g. to bi-Lipschitz nonembedding theorems. We will give an overview of the subject, including a description of some recent progress.