Regularity of optimal transportation maps on multiple products of spheres

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Optimal transportation seeks a map which transports a given mass distribution to another while minimizing the transportation cost. Existence and uniqueness of optimal transportation maps is well known on Riemannian manifolds where the transportation cost of moving a unit mass is given by the distance squared function. However, regularity (such as continuity and smoothness) of such maps is much less well known, especially beyond the case of the round sphere and its small perturbations. Moreover, if the manifold has negative curvature somewhere, there are discontinuous optimal maps even between smooth mass distributions.

In this talk, we explain a regularity result for optimal maps on products of multiple round spheres of arbitrary dimension and size — the first such result for non-flat Riemannian manifolds whose curvature is not strictly positive. This is joint work with Alessio Figalli and Robert McCann.