Homogeneous Ricci flows and solitons

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Abstract: In the spirit of the DeTurck's trick, we shall describe an ODE for a curve of Lie brackets which is equivalent in a natural and specific sense to the Ricci flow starting at any homogeneous Riemannian manifold, but which has proved to be much more friendly, at least in some particular cases (as, for instance, nilmanifolds). Indeed, it is easy to prove from this perspective (already considered by Guzhvina and Payne) that for any simply connected nilmanifold, all the solutions to the Ricci flow are Type-III, and even with a constant which only depends on the dimension. Certain convergence results have also been obtained for nilmanifolds by using this approach.

Concerning Ricci solitons, we will define algebraic solitons on homogeneous spaces by generalizing the concept of nilsoliton and give an idea of the proof of the following: any example of an algebraic soliton which is not a solvmanifold would give rise to a counterexample to the long standing Alekseevskii's conjecture on Einstein homogeneous manifolds of negative scalar curvature.