Global regularity of the reflector problem

Jiakun Liu
Princeton University

In this talk, we study a reflector system which consists of a point light source, a reflecting surface and an object to be illuminated. Due to its practical applications in optics, electro-magnetics, and acoustics, it has been extensively studied during the last half century. This problem involves a fully nonlinear partial differential equation of Monge–Ampere type, subject to a nonlinear second boundary condition. In the far field case, it is related to the reflector antenna design problem. By a duality, namely a Legendre type transform, Xu-Jia Wang has proved that it is indeed an optimal transportation problem. Therefore, the regularity results of optimal transportation can be applied. However, in the general case, the reflector problem is not an optimal transportation problem and the regularity is an extremely complicated issue. In this talk, we will give necessary and sufficient conditions for the global regularity and briefly discuss their connection with the Ma–Trudinger–Wang condition in optimal transportation.