“Equations are just the boring part of mathematics. I attempt to see things in terms of geometry.”
— Stephen Hawking

Description

In many cases, nature finds optimal solutions to geometric problems. For example, among all possible shapes spanning a loop of wire, a soap film always has least area. The hexagonal packing system used by bees in their honeycombs is the most efficient (least perimeter) way to divide a plane into equal areas. Remarkably, these and other shapes found in physical or biological systems also appear in abstract mathematics as solutions to certain optimization problems. Many of these forms also frequently appear in art and architecture, and are regarded as beautiful by many cultures.

These phenomena raise several interesting questions. What does it mean to have a “best form?” Indeed, is the concept of “best” even well defined in this context? Can mathematical reasoning prove that something is optimal in a precise sense? What roles do evolution and physical principles play in finding such forms? How and why do various cultures integrate such forms into their patterns of artistic expression?

This course will examine some of the attempts of science to provide rigorous explanations of optimal geometry. Historically, such attempts had both empirical and synthetic (i.e. philosophical or theological) motivations. Some even incorporated ideas that today might provocatively be labeled “intelligent design.” The course will also explore how artists and architects incorporate geometric principles in their designs.

The main focus of the course will investigate what insights might be gained by looking at nature and art through the lens of mathematics — specifically, by investigating how our ways of understanding mathematics and the natural world inform and interact with each other. In accord with the *Modes of Reasoning* rubric, we will devote particular attention to the roles of geometric reasoning and mathematical epistemology in making our explanations intellectually rigorous. We will do this through four “case studies,” organized around the following topics:

1. Optimality and Minimal Surfaces (can optimality be proved with mathematical rigor?);
2. Pattern and Abstraction (why does nature repeat itself?);
3. Evidence and Proof (what convinces a mathematician that something is true?); and
4. Symmetries and their Structure (why do symmetries form and break?).

Sources

The course will draw on multiple sources, which will be excerpted and included in a custom course packet. Some of the sources currently under consideration are as follows:

- *The Parsimonious Universe: Shape and Form in the Natural World*, by Stefan Hildebrandt and Anthony Tromba;
Assessments

The course grade will be based on the following components:

- Exams (64%). There will be four in-class exams, each worth 16% of the total grade.
- Homework (20%). Each topical unit in the course will be accompanied by a small project.
- Class participation (16%). This component will assess participation in class discussions and completion of daily “minute papers” — brief impressionistic paragraphs written in response to questions raised during class discussion.

About the professor

Dan Knopf is an Associate Professor and the Graduate Adviser in the Department of Mathematics. He joined the University of Texas at Austin in fall of 2004. He received a National Science Foundation CAREER award in 2006, and a College of Natural Sciences Teaching Excellence Award in 2012. He is an active researcher in geometric analysis — in particular, the use of geometric evolution equations to find and classify canonical or optimal geometries. He is author or coauthor of over twenty-five scholarly publications, including five books. His nonacademic interests include running, gardening, spoiling two cats, and rooting for the Longhorns.