

# Gradient Ricci solitons: the comparison geometry point of view

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**Abstract:** In this talk, instead of approaching Ricci solitons as special solutions to the Ricci flow, we will begin by studying them as the natural generalization of Einstein manifolds coming from metric measure theory. Interestingly, this viewpoint appears as far back as the work of Lichnerowicz in the 1970s and has also been studied extensively in a somewhat different direction by Bakry and Emery and their collaborators. (It also appears in Perelman's work and has been generalized to give a definition of Ricci curvature for non-smooth spaces in the work of Lott–Villani and Sturm.)

Once we have motivated this viewpoint, we will discuss some classification results and open questions for gradient Ricci solitons which arise. One of these results (perhaps unfortunately from the viewpoint of this workshop!) will be that there are no non-trivial homogeneous gradient Ricci solitons. However, we also ask whether existing constructions of Ricci solitons on homogeneous spaces can be modified to give (non-homogeneous) gradient Ricci solitons which have certain curvature properties.

Finally we will attempt to come full circle and describe some possible applications of these ideas to general Ricci flows.