Some fish species in AFMA managed fisheries

Yellowfin and Skipjack Tuna

Yellowfin Tuna photo in large format (217kb)

John Dory

John Dory photo in large format (305kb)

Tiger Prawn

Tiger Prawn photo in large format (171kb)

Scallops

Scallops photo in large format (387kb)

Yellowfin Tuna

Yellowfin Tuna photo in large format (190kb)

Patagonian Toothfish

Patagonian Toothfish photo in large format (350kb)
Proper management of fish resources is essential for survival of the (piscine) species and for the future of humans economically dependent on fishing. Here we’re going to take a simple model for harvesting of fish. It’s a combination discrete-continuous model. The advantage to have a discrete component is that one could go in to do a stochastic version, and check long-term viability, by varying parameters over the years and stochastically.

In this model, we have a population of fish; there are \( P_n \) in year \( n \), and, if there were no fishing, there would be \( P_{n+1} \) in year \( n+1 \). The difference equation \( P_{n+1} = f(P_n) \) tells us how many hatchlings survive to become adults. Traditionally \( f \) is called the recruitment function, because it tells us how many juveniles are recruited into adulthood. We’ll start by modeling that, with the so-called Beverton-Holt stock recruitment equation.

In the Beverton-Holt model, hatchlings \( h \) at time \( t \) compete with each other for survival over the course of a year. The competition might be for safe hiding places from predators, or it might be simply for food. In either case, we model the competition as

\[
\frac{dh}{dt} = -[m_1 + m_2 h(t)]h(t)
\]

1) This is easy enough to solve; it’s a Bernoulli equation or you can use partial fractions to integrate it. In either case, solve this for \( h \) as a function of \( t \).

2) Now, for initial conditions. If the fish have a reproductive rate of \( r \), we expect \( h(n) = rP_n \), and we also expect \( h(n+1) = P_{n+1} \). Using these conditions, solve for \( f(P_n) \).

3) Now that our fish are all grown up, it’s time to harvest them: we take away a yield of \( Y_n \) of the \( P_{n+1} \) fish, leaving \( P_{n+1} = f(P_n) - Y_n \). We’re eventually going to talk about equilibrium, at which \( P = f(P) - Y \), and we want to decide what is the maximum \( Y \) we can take. The problem is, that depends on the number of fish \( P \), and that in turn is rather hard to count. What we’ll try and do is eliminate \( P \) from this equation, in favor of a variable we can count: \( E \), the amount of energy expended per unit time during a fishing season of length \( \tau \). There’s no reason to vary \( E \), so we get a nice equation for fishing season: the number of fish caught is proportional to the energy expended times the number of fish, leading to a decrease in the fish population as:

\[
\frac{dP}{dt} = -qEP
\]

Solve this over the length of the fishing season, that is from \( t = t_0 \) to \( t = t_0 + \tau \), using \( P(t_0) = f(P_n) \) and \( P(t_0 + \tau) = f(P_n) - Y_n \) (kinda by definition of \( Y_n \) as the yield).

4) Now assume we are at equilibrium, and that the recruitment function is given by the Beverton-Holt model. Show that

\[
Y = K(R_0e^{-q\tau E} - 1)(e^{q\tau E} - 1)
\]

5) Maximize \( Y \) with respect to \( E \).