1)(20 points) Let \( f(x) \) be defined as

\[
f(x) = \frac{x^2 - x - 2}{|x^2 + x|}
\]

a) Factor a *signum* function from \( f \).

b) Use a) to write \( f \) as a piecewise defined function.

c) Use left, right limits to determine if \( f \) has a jump discontinuity at \(-1\).

\[
a) \quad f = \frac{(x+1)(x-2)}{|x(x+1)|} = \frac{x-2}{1x1} \quad \frac{x+1}{1x+1}
\]

\[
b) \quad f = \begin{cases} \frac{x-2}{1x1} & \text{if} \quad x > -1 \\ -\frac{x-2}{1x1} & \text{if} \quad x < -1 \\ \text{lim} & \text{if} \quad x = -1 
\end{cases}
\]

\[
c) \quad \lim_{x \to -1^-} f = \lim_{x \to -1^-} \frac{-x^2}{1x1} = -\frac{(-2)}{1-1} = 3
\]

\[
\lim_{x \to -1^+} f = \lim_{x \to -1^+} \frac{x^2}{1x1} = \frac{-2}{1-1} = -3
\]

\(3 \neq -3\) so \( f \) lies a jump at \( x = -1 \)
2) (20 points) \( C f(x) = \frac{x^4 - x - 1}{x^2 + x} \).

a) Find a \( g \) to which \( f \) is asymptotic.

b) Show \( f \) is asymptotic to \( g \) by computing a limit.

\[ f(x) = \frac{x^2 + x}{x^4 - x - 1} \]

\[ \frac{d}{dx} \left( \frac{x^2 + x}{x^4 - x - 1} \right) \]

\[ = \frac{(x^4 - x - 1)(2x + 1) - (x^2 + x)(4x^3 - 1)}{(x^4 - x - 1)^2} \]

\[ = \frac{-2x^2 - 1}{x^2 + 1} \]

5) \( \lim_{x \to \pm \infty} f(x) = \lim_{x \to \pm \infty} \frac{-2x - 1}{x^2 + 1} = \lim_{x \to \pm \infty} \frac{-2 - \frac{1}{x^2}}{1 + \frac{1}{x^2}} = 0 \)
3) (20 points) Use freezing to sketch the graph of \( y = x(1 + x)^{\frac{2}{3}} \).

Show your work beneath here

**Schematic Points:**
- \( x = 0 \), \( x = -1 \)

- \( x = 0 \)
  \[ y = x(1 + 0)^{\frac{2}{3}} = x \]

- \( x = -1 \)
  \[ y = (-1)(1-1)^{\frac{2}{3}} = - (1 + x)^{\frac{2}{3}} \]

Use the space beneath here to draw your graph.
4)(40 points) Let

\[ f(x) = x^3(x^2 - 1)^2 \]

a) Use the product and chain rules to differentiate

b) Simplify

c) Find all critical points

\[ f'(x) = 2x^2(x^2 - 1)^2 + x^2 2(x^2 - 1)(2x) \]

\[ f'(x) = x^2(x^2 - 1) \left[ 3(x^2 - 1) + 4x^2 \right] \]

\[ = x^2(x^2 - 1)(7x^2 - 2) \]

\[ x = 0, \pm 1, \pm \sqrt{\frac{2}{7}} \]
1) (30 points) Let \( f(x) \) be defined as

\[
 f(x) = \frac{x^2 + x}{|x^2 - x - 2|}
\]

a) Factor the \textit{signum} function from \( f \).

b) Use a) to write \( f \) as a piecewise defined function.

c) Use left, right limits to determine if \( f \) has a jump discontinuity at \(-1\).

\begin{align*}
\text{a)} \quad f &= \frac{x(x+1)}{|(x+1)(x-2)|} = \frac{x}{|x-2|} \left[ \frac{x+1}{(x+1)} \right] \\
\text{b)} \quad f &= \begin{cases} \\
\frac{x}{|x-2|} & \text{if} \ x > -1 \\
\frac{-x}{|x-2|} & \text{if} \ x < -1 \\
\text{dne} & \text{if} \ x = -1
\end{cases}
\end{align*}

\begin{align*}
\text{c)} \quad \lim_{x \to -1^+} f &= \lim_{x \to -1^+} \frac{x}{|x-2|} = \frac{-1}{1-3} = -\frac{1}{2} \\
\lim_{x \to -1^-} f &= \lim_{x \to -1^-} \frac{-x}{|x-2|} = \frac{-(-1)}{1-3} = \frac{1}{2}
\end{align*}

\[-\frac{1}{2} \neq \frac{1}{2}\] so \( \lim_{x \to -1} f \) dne

and \( f \) does have a jump at \( x = -1 \).
2) (20 points) Let

\[ f(x) = \frac{x^3 - x^2 - 1}{x^2 + x} \]

a) Find a \( g \) to which \( f \) is asymptotic.
b) Show \( f \) is asymptotic to \( g \) by computing a limit.

\[
\frac{x^2 + x}{x^2 - x^2 - 1} \quad \frac{x - 2}{x^2 - x^2 - 1}
\]

\[
- (x^2 + x^2)
\]

\[
-2 x^2 - 1
\]

\[
(-2 x^2 -2x)
\]

\[
2x - 1
\]

\[ a) \quad \lim_{x \to 1} = x - 2 \]

\[ b) \quad \lim_{x \to \pm\infty} \left[ f - g \right] = \lim_{x \to \pm\infty} \left[ \frac{2x - 1}{x^2 + x} \right] \]

\[
= \lim_{x \to \pm\infty} \left[ \frac{2x - 1}{1 + \frac{x}{x}} \right] = \frac{0}{1} = 0 \downarrow \]
3) (15 points) Use freezing to sketch the graph of \( y = x^2(x - 1)^{\frac{2}{3}} \).

Show your work beneath here

Freezing points: \( x = 0, 1 \)

When \( x = 0 \) is small \( x^2 (x-1)^{2/3} \approx x^2 \)

When \( x = 1 \) is small \( (x-1)^{2/3} \approx -(x-1)^{2/3} \)

Use the space beneath here to draw your graph
4) (35 points) Use the product and chain rule to differentiate, simplify, and find all critical points:

\[ y = x(x^2 - 1)^2 \]

\[ y' = \left[ (x^2 - 1)^2 + x \cdot 2(x^2 - 1)(2x) \right] \quad \text{or} \quad y' = (x^2 - 1) \left[ (x^2 - 1) + 4x^2 \right] + (x^2 - 1) \left[ 5x^2 - 1 \right] \]

\[ y' = 0 \quad \text{when} \quad x = \pm \frac{1}{\sqrt{5}}, \quad x = \frac{1}{\sqrt{5}} \]

\[ CP: \quad x = \pm \frac{1}{\sqrt{5}}, \quad x = \frac{1}{\sqrt{5}} \]
1)(20 points) Let \( f(x) \) be defined as

\[
f(x) = \frac{x^2 - x - 2}{|x^2 + x|}
\]

a) Factor a signum function from \( f \).

b) Use a) to write \( f \) as a piecewise defined function.

c) Use left, right limits to determine if \( f \) has a jump discontinuity at \(-1\).

\[a) \quad f = \frac{(x+1)(x-2)}{|x(x+1)|} = \frac{x-2}{1 \times 1} \quad \frac{x+1}{1 \times 1}\]

\[b) \quad f = \begin{cases} \frac{x-2}{1 \times 1} & \text{if} \quad x > -1 \\
-\frac{x-2}{1 \times 1} & \text{if} \quad x < -1 \\
\text{undefined} & \text{if} \quad x = -1\end{cases}\]

\[c) \quad \lim_{x \to -1^-} f = \lim_{x \to -1^-} \frac{-x-2}{1 \times 1} = \frac{-3}{1 \times 1} = -3\]

\[\lim_{x \to -1^+} f = \lim_{x \to -1^+} \frac{x-2}{1 \times 1} = \frac{-3}{1 \times 1} = -3\]

\[z \neq -3 \quad \text{so} \quad f \text{ has a jump at } x = -1\]
2) (15 points) Compute the limit:

\[
\lim_{{x \to -1}} \frac{{(x^3 + 6x^2 + 11x + 6)}}{{(x^2 - x - 2)}} = \frac{{-1 + 6 - 11 + 6}}{1 + 1 - 2} = 0
\]

Factor and cancel: \(x - (-1) = x + 1\)

\[
\begin{align*}
    x^2 - x - 2 &= (x + 1)(x - 2) \\
    x + 1 &\quad \frac{x^2 + 5x + 6}{(x + 1)(x - 2)} \\
    \downarrow &\quad \frac{x^2 + 11x + 6}{(x + 1)(x - 2)} \\
    - &\quad \frac{3x^2 + 11x + 6}{(x + 1)(x - 2)} \\
    - (5x^2 + 5x) &\quad \frac{6x + 6}{(x + 1)(x - 2)} \\
    &\quad \frac{6x + 6}{0} \\
\end{align*}
\]

Therefore:

\[
\lim_{{x \to -1}} f = \lim_{{x \to -1}} \frac{{x^2 + 5x + 6}}{x - 2} = \frac{{-1 - 5 + 6}}{-1 - 2} = \frac{{-2}}{-3} = -\frac{2}{3}
\]
3) (20 points) Use freezing to sketch the graph of \( y = x(1 + x)^{\frac{2}{3}} \).

Show your work beneath here.

**Slope Points:**
- \( x = 0, \ x = -1 \)

\[ \begin{align*}
  x = 0 & \quad y = x(1 + 0)^{\frac{1}{3}} = x \\
  x = -1 & \quad y = (-1)(1 + (-1))^{\frac{1}{3}} = -1
\end{align*} \]

Use the space beneath here to draw your graph.
4) (45 points) Let 

\[ f(x) = \frac{(x+1)^2}{x^3} \]

a) Use the product and chain rules to differentiate
b) Simplify
c) Find all critical points

\[ f'(x) = \frac{2(x+1)x^{2/3} - \frac{2}{3}(x+1)^2 x^{-2}}{x^{4/3}} \]

\[ = \frac{2(x+1)\left[3x - (x+1)\right]}{3x^{1/3}} \]

\[ y' = 0 \text{ when } x = -1, \frac{1}{2} \]

\[ y' \text{ due to } \theta = 0 \]

\[ \frac{2}{D_{1,5}} = \frac{1}{D} \text{ due } \]

\[ CP: x = -1, x = \frac{1}{2} \]
1) (30 points) Let $f(x)$ be defined as 

$$f(x) = \frac{x^2 + x}{|x^2 - x - 2|}$$

a) Factor a *signum* function from $f$.
b) Use a) to write $f$ as a piecewise defined function.
c) Use left, right limits to determine if $f$ has a jump discontinuity at -1.

c) 

$$\lim_{x \to -1^+} f = \lim_{x \to -1} \frac{x}{|x-2|} = \frac{-1}{1-3} = -\frac{1}{2}$$

$$\lim_{x \to -1^-} f = \lim_{x \to -1} \frac{-x}{|x-2|} = \frac{-(-1)}{1-3} = \frac{1}{2}$$

$$-\frac{1}{2} \neq \frac{1}{2}$$

So, $\lim_{x \to -1} f \neq \lim_{x \to -1} f$ due

and $f$ does have a jump at $x = -1$
2) (20 points) Compute

\[ \lim_{x \to -1} \frac{(x^3 - x^2 + 2)}{(x^2 + x)} = \frac{-1 - 1 + 2}{-1} = 0 \]

Factor and cancel out \( x - (-1) = x + 1 \) from numerator and denominator.

\[ x^2 + x = x(x + 1) \checkmark \]

\[ x + \frac{x^3 - 2x + 2}{x^3 - x^2 + 2} \]
\[ = \frac{-x^2 + 2}{-2x^2 - 2x} \]
\[ = \frac{2x + 2}{2x + 2} \]
\[ = 1 \]

\[ \ln f = \lim_{x \to -1} \frac{x^2 - 2x + 2}{x} = \frac{1 + 2 + 2}{-1} = 5 \]
3) (15 points) Use freezing to sketch the graph of \( y = x^2(x - 1)^{\frac{2}{3}} \).

Show your work beneath here

Freezing points: \( x = 0, 1 \)

Near \( x = 0 \) it is like \( x^2 (-1)^{\frac{2}{3}} = x^2 \)

Near \( x = 1 \) it is like \( 1^2 (x-1)^{\frac{2}{3}} = (x-1)^{\frac{2}{3}} \)

Use the space beneath here to draw your graph
4) (35 points) Use the quotient and chain rule to differentiate, simplify, and find all critical points:

\[ y = \frac{(x + 1)^2}{x^{\frac{1}{3}}} \]

\[ y' = \frac{2(x + 1)x^{\frac{1}{3}} - (x + 1)^2 \cdot \frac{1}{3} x^{-\frac{2}{3}}}{(x^{\frac{1}{3}})^2} \]

\[ = \frac{2(x + 1)x^{\frac{1}{3}}}{1} - \frac{(x + 1)^2}{3 x^{\frac{2}{3}}} \]

\[ = \frac{(x + 1) \left[ 6x - (x + 1) \right]}{3 x^{\frac{2}{3}}} \]

\[ = \frac{(x + 1)(5x - 1)}{3 x^{\frac{2}{3}}} \]

\[ y' = 0 \] when \( x = -1, \) \( x = \frac{1}{5} \).

\( y' \) is undefined when \( x = 0 \).

\[ f(0) = \frac{(0 + 1)^2}{0^{\frac{1}{3}}} = \infty \] due to \( x = 0 \).

\( CP: \ x = -1, \frac{1}{5} \).