Freezing: One For You

Use freezing to guess where the critical points might be. Then differentiate, simplify, and find all critical points to check your answer.

\[ a) \quad y = x(1 - x) \]
\[ b) \quad y = x^2(1 + x) \]
\[ c) \quad y = x^2\sqrt{1 - x} \]
\[ d) \quad y = x(1 - x^2) \]
a) \( y = x(1-x) \). The zeroes are \( x = 0, x = 1 \).

So you will freeze at these.

When \( x = 0 \), freeze the \( 1-x \) to \( 1-0 = 1 \).

So \( y = x(1) = x \).

When \( x = 1 \), freeze the \( x \) to 1 so

\[ y = 1(1-x). \]

Critical point in between:

Use der to locate exactly

\[ y' = (x)(1-x)' + x(1-x)' = x((-x)) + x(1) \]

\[ = 1 - x - x = 1 - 2x. \]

\[ y' = 0 \] when \( x = \frac{1}{2} \). Thus \( y \)

in between \( x = 0 \) and \( x = 1 \).
b) $y = x^2 (1 + x)$

Intercepts: $c = 0$, $x = -1$.

- At $x = 0$, freeze $1 + x$ to 1+0 so $y = x^2(1) = x^2$.
- At $x = -1$, freeze $x^2$ to $(-1)^2 = 1$ so $y = 1(x+1)$.

$cP$ between zero and $-1$ use $y'$ to locate.

$y' = (x')' (1 + x) + x^2 (1 + x)'$
$= 2x (1 + x) + x^2(1) = 2x + 2x^2 + x^2$
$= 2x + 3x^2 = x(2 + 3x)$

$y' = 0$ when $x = 0$, $x = -2/3$.

$-2/3$ is between zero and $-1$. 

1. \( y = x^2 \sqrt{1-x} \) 
   - Intercepts at \( x = 0 \), \( x = 1 \)
   - At \( x = 0 \), \( y = 0 \)
   - At \( x = 1 \), \( y = \sqrt{1-x} \)

\[ f'(x) = 2x \sqrt{1-x} + \frac{x^2}{2 \sqrt{1-x}} \left(1 - 2x \right) \]

\[ = 2x \sqrt{1-x} - \frac{x^2}{2 \sqrt{1-x}} = \frac{4x(1-x) - x^2}{2 \sqrt{1-x}} \]

\[ = \frac{4x - 4x^2 - x^2}{2 \sqrt{1-x}} = \frac{4x - 5x^2}{2 \sqrt{1-x}} \]

\[ = \frac{x(4 - 5x)}{2 \sqrt{1-x}} \]

Critical points: \( x = 0 \), \( x = \frac{4}{5} \), \( x = 1 \)

Note: \( \frac{4}{5} \) is between 0 and 1.
\( d) \quad y = x^2(1-x^2) = x(x-1)(1+x). \)

\[ x = 0 \quad \text{free} \quad (1-x)(1+x) \quad \text{to} \quad (1+0)(1-0) = 1 \quad \eta = x^2 \]

\[ x = 1 \quad \text{free} \quad x^2(1+x) \quad \text{to} \quad 1^2(1+1) = 2 \quad \eta = 2(1-x) \]

\[ x = -1 \quad \text{free} \quad x^2(1-x) \quad \text{to} \quad (-1)^2(1-(-1)) = 2 \quad \eta = 2(1+x) \]

\[ y' = 2x(-y^2) + y^2(-2x) = 2x[y^2 - y^2] = 2x(1-y^2) \quad y' = 0 \quad \text{when} \ x = 0, \]

\[ x = \pm \frac{1}{\sqrt{2}} \]

\(-\frac{1}{\sqrt{2}} \text{ is between } -1 \text{ and } 0\)

\(+\frac{1}{\sqrt{2}} \text{ is between } 0 \text{ and } 1\).