1) A volume sits above the area bounded by the curves $y = x^2$, $y = 0$ in the $xy$-plane. Each $x$ cross section is a rectangle with base touching the ends of the curves and height twice the base. If $-1 \leq x \leq 1$, what is the volume?
   a) Sketch the region in the $xy$ plane.
   b) What is the area of a cross-section at $x$?
   c) Write an integral for the volume.
   d) Find the value of the integral.

2) A volume sits above the triangle in the $xy$-plane. The triangle sides are defined by the equations $y = x; y = -x; x = 1$. Each $x$ cross section is a half-circle, with diameter touching the ends of the $y = \pm x$ curves. What is the volume?
   a) Sketch the region in the $xy$ plane.
   b) What is the area of a cross-section at $x$?
   c) Write an integral for the volume.
   d) Find the value of the integral.

3) What is the volume of the object $y^2 + z^2 = x; 0 \leq x \leq 1$?
b) \( A(x) = \text{area of rectangle} \times \) = \( 2b^2 \) \\
height = x \times \text{base} = \text{so} \quad A = 2b^2.

box runs from one curve to another between
\( y = 0 \) to \( y = x^2 \). So \( \text{area} = x^2 - 0 = x^2 \) \\
\( A(x) = 2b^2 = 2(x^2)^2 = 2x^4 \)

c) \( V = \int_a^b A(x) \, dx \quad \alpha = \text{left-most} \quad x = -1 \) \\
\( b = \text{right-most} \quad x = +1 \) \\
\( V = \int_{-1}^{1} 2x^4 \, dx \)

d) \( V = \int_{-1}^{1} 2x^4 \, dx \) is even on a symmetric interval \\
\( = 2 \int_{0}^{1} x^4 \, dx = 4 \int_{0}^{1} x^4 \, dx = 4 \int_{0}^{1} x^5 \, dx \)
\( = \frac{4}{5} [ x^5 ]_{0}^{1} = \frac{4}{5} [1 - 0] = \frac{4}{5} \)

the other problems are for you to do in TA section.
14. Volumes

A volume lies above the equilateral triangle \( A \) in the xy plane, defined by 
\[ y = x, \quad y = -x, \quad x = 1. \]
Each x-section is a half-circle with diameter lying touching the ends of the triangle. What is the volume?

b) Sketch the region in the xy plane

\[ y = x \]
\[ y = -x \]
\[ x = 1 \]

c) Set up the volume by sections

\[ V = \int_{0}^{1} \pi (x) \, dx \]

\[ a = 0, \quad b = 1 \]

d) Start filling in: \( a, b, r \). The picture in (a) shows \( a = 0, b = 1 \)

e) Find \( A(x) \). It's half a circle, so \( A(x) = \frac{\pi r^2}{2} \).

But what is the radius for each \( x \)?

So, back to the picture.

\[ \frac{1}{2} \text{diameter} = \frac{1}{2} \text{radius} \]
2) What is the volume of the object 
\[ y^2 + z^2 = x, \quad 0 \leq x \leq 1 \]?

This is a cylinder: 
\[ V = \pi \int_0^1 A(x) \, dx \] where we were flat-out given \( a = 0, \, b = 1 \).

What is \( A(x) \)? Well, for each \( x \) we have 
\[ y^2 + z^2 = x. \] That is a circle of radius \( \sqrt{x} \).

Then 
\[ A(x) = \pi \left( \text{radius} \right)^2 = \pi \sqrt{x}^2 = \pi x. \]

So 
\[ V = \int_0^1 \pi x \, dx = \pi \left[ \frac{x^2}{2} \right]_0^1 = \frac{\pi}{2} \]