1) (35 points) Let

\[ f(x) = \frac{1}{\sqrt{1 - 2x}} = 1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3 + \ldots \]

a) Find \( S_3 \), \( b_4 \).

b) Use \( S_3 \) to compute \( \frac{1}{\sqrt{1.2}} \).

c) How large can error be?

d) A calculator gives \( \frac{1}{\sqrt{1.2}} = .91287092 \ldots \). Round correctly to five decimal places.

e) How large can error be?

\[ S_3 = 1 + x + \frac{3}{2}x^2 \]

\[ b_4 = \left| \frac{5}{2}x^3 \right| \]

b) \[ \frac{1}{\sqrt{1.2}} = \frac{1}{\sqrt{1.2}} \]

\[ -2x = 1.2 \]

\[ x = -0.6 \]

\[ S_3 = 1 + (-0.6) + \frac{3}{2}(-0.6)^2 = 1 - 0.6 + 1.5 \times 10^{-2} \]

\[ \cdot 3 = 0.915 \]

c) \( \text{error} \leq |b_4| = \left| \frac{5}{2}x^3 \right| = \left| \frac{5}{2}(-10^{-1})^3 \right| = 5/2 \times 10^{-3} = 2.5 \times 10^{-3} = 0.0025 \)

\[ \text{error} < 0.025 \]

d) \( .91287092 \rightarrow .91287 \) five dp

e) \( \text{error} < 0.0022 \)
2) (20 points) Use Taylor series (not power series) to find $T_5$ for $f(x) = \ln(1-x), \quad a = 0$. Show work, and simplify your answer.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$f^{(k)}(x)$</th>
<th>$f^{(k)}(0)$</th>
<th>$f^{(k)}(0) \times \frac{x^k}{k!}$</th>
<th>$\sum_{n=0}^{\infty} a_n x^n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\ln(1-x)$</td>
<td>0</td>
<td>$0 \times 0!$</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>$(1-x)^{-1}$</td>
<td>-1</td>
<td>$-x^1/1!$</td>
<td>$-x$</td>
</tr>
<tr>
<td></td>
<td>$-1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$-(1-x)^{-2}$</td>
<td>-1</td>
<td>$-x^2/2!$</td>
<td>$-x^2/2$</td>
</tr>
<tr>
<td></td>
<td>$-2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$-2(1-x)^{-3}$</td>
<td>-2</td>
<td>$-2 \times x^3/3!$</td>
<td>$-x^3/3$</td>
</tr>
<tr>
<td></td>
<td>$-6$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$\frac{1}{2}(1-x)^{-4}$</td>
<td>-3</td>
<td>$-3 \times x^4/4!$</td>
<td>$-x^4/4$</td>
</tr>
<tr>
<td></td>
<td>$-12$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$-6(1-x)^{-5}$</td>
<td>-4</td>
<td>$-4 \times x^5/5!$</td>
<td>$-x^5/5$</td>
</tr>
<tr>
<td></td>
<td>$-30$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$T_5 = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5}$$
3) (20 points) Does the following converge or diverge?

\[
\sum \frac{(k+1)^{2k}}{(\ln k)^k} (-1)^{\frac{k \cdot \ln k}{k}} \left( \frac{(k+1)^{2k}}{(\ln k)^k} \right)^{\frac{1}{k}}
\]

\[
|a_k|^{\frac{1}{k}} = |(-1)^{k+1}|^{\frac{1}{k}} \left( \frac{(k+1)^{2k}}{(\ln k)^k} \right)^{\frac{1}{k}}
\]

\[
= \frac{(k+1)^2}{\ln k}
\]

\[
\lim_{k \to \infty} |a_k|^{\frac{1}{k}} = \lim_{x \to \infty} \frac{(x+1)^2}{\ln x} = \infty
\]

\[
= \lim_{x \to \infty} \frac{2(x+1)}{\frac{1}{x}} = \lim_{x \to \infty} 2x(x+1) = \infty
\]

\[
2- \infty > 1 \text{ so } \sum a_k \text{ diverges by root.}
\]
4) (25 points) Let

\[ f(x) = \int \frac{x}{(1 - x)^2} \, dx \quad f(0) = 0 \]

Use the series for \(1/(1 - x)\), to find the power series centered at 0.

a) Write the first three non-zero terms, simplified.

b) Write the series using summation notation.

\[
\frac{1}{(1-x)^2} = \frac{d}{dx} \left[ \frac{1}{1-x} \right] = \frac{d}{dx} \left[ 1 + x + x^2 + x^3 + \cdots \right] = \frac{d}{dx} \left[ \sum_{k=0}^{\infty} x^k \right] = \sum_{k=0}^{\infty} k \cdot x^{k-1} = 1 + 2x + 3x^2 + \cdots
\]

\[
\text{for } k = 0, 1, 2, \ldots
\]

\[ \frac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + \cdots = \sum_{k=0}^{\infty} k \cdot x^k \]

\[
\int \frac{x}{(1-x)^2} \, dx = \frac{x^2}{2} + \frac{2}{3} x^3 + \frac{3}{4} x^4 + \cdots = \sum_{k=0}^{\infty} \frac{k}{(k+1)} x^{k+1} + C
\]

\[ f(0) = 0 \quad \text{so} \quad 0 + 0 + \cdots + C = 0 \quad \text{as} \quad C = 0
\]