1) (20 points) Let \( f(x, y) = y/(x^2 + y^2) \). Compute and simplify:

\[
\frac{\partial f}{\partial x} = y \frac{\partial}{\partial x} \left( \frac{1}{x^2 + y^2} \right) = -y \left( x^2 + y^2 \right)^{-2} \cdot 2x = -\frac{2xy}{(x^2 + y^2)^2}
\]

\[
\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \left( \frac{y}{x^2 + y^2} \right) = \frac{1}{x^2 + y^2} - \frac{2y}{(x^2 + y^2)^2} = \frac{x^2 - y^2}{(x^2 + y^2)^2}
\]
2) (25 points) Let \( f(x, y) = \sqrt{1 - x^2 - y^2}, P = P(0, \frac{1}{2}). \)

a) Sketch the surface; use about a third to the page. Locate \( P, f(P). \)

b) In the \( xy \) plane, sketch the trace \( z = f(x, \frac{1}{2}). \) Locate \( P, f(P). \)

c) On the surface, sketch the trace \( z = f(x, \frac{1}{2}). \)

\[
f(\theta) = \sqrt{1 - \cos^2 \theta} = \frac{\sqrt{3}}{2}
\]

\[
y) \quad 3 = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}
\]
3) (25 points) Let \( z = f(x, y); \ x = u \cosh(v), \ y = u \sinh(v). \)

a) Draw the tree for \( f. \)

b) State the chain rule for \( \frac{\partial f}{\partial v}. \)

c) Compute \( \frac{\partial f}{\partial v} \) directly.

\[
\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial v}
\]

\[
x = u \cosh(v), \quad y = u \sinh(v)
\]

\[
\frac{\partial x}{\partial v} = u \sinh(v), \quad \frac{\partial y}{\partial v} = u \cosh(v)
\]

\[
\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x} \cdot u \sinh(v) + \frac{\partial f}{\partial y} \cdot u \cosh(v)
\]

\[
= y \frac{\partial f}{\partial x} + x \frac{\partial f}{\partial y}
\]

d) Let \( f(x, y) = x^2 - y^2. \) Use c) to show \( f \) has no \( v \)'s in it.

\[
\frac{\partial f}{\partial x} = 2x, \quad \frac{\partial f}{\partial y} = -2y
\]

\[
\frac{\partial f}{\partial v} = y \frac{\partial f}{\partial x} + x \frac{\partial f}{\partial y} = 0
\]
4) (30 points) Let \( R \) be the region \( R = \{(x, y) \mid x^2 + y^2 \leq 1; \ x \leq 0; \ y \leq 0\} \).

a) Sketch the region \( R \) (use about an eighth of the page).

b) Write the region as a type two region.

c) Compute the integral

\[
\int_R \int \sqrt{x^2 + y^2} \ dA
\]

\[
R = \left\{ (x, y) \mid -1 \leq y \leq 0, \ -\sqrt{1-y^2} \leq x \leq 0 \right\}
\]

\[
\sqrt{x^2 + y^2} \Rightarrow \sqrt{r^2} = r
\]

\[
dx \ dy \rightarrow r \ dr \ d\theta
\]

\[
R \rightarrow \left\{ (r, \theta) \mid 0 \leq r \leq 1, \ \pi \leq \theta \leq \frac{3\pi}{2}\right\}
\]

\[
\int_\pi^{3\pi/2} \int_0^1 r^2 \ dr \ d\theta
\]

\[
\int_\pi^{3\pi/2} \int_0^1 r \ dr \ d\theta = \left[ \frac{r^3}{3} \right]_0^1 = \frac{1}{3}
\]

\[
\int_\pi^{3\pi/2} \int_0^1 \frac{1}{3} \ d\theta = \frac{1}{3} \int_\pi^{3\pi/2} \ d\theta = \frac{1}{3} \left[ \frac{3\pi}{2} - \pi \right] = \frac{1}{3} \left[ \frac{\pi}{2} \right]
\]

\[
\approx \frac{\pi}{6}
\]
1) (20 points) Let \( f(x, y) = \frac{x^2}{x + y} \). Compute and simplify:

\[
\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}
\]

\[
\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \left( \frac{x^2}{x + y} \right) = \frac{x^2 \frac{\partial}{\partial x} (x + y) - (x + y) \frac{\partial}{\partial x} x^2}{(x + y)^2}
\]

\[
= \frac{2x(x + y) - x^2}{(x + y)^2}
\]

\[
= \frac{2x^2 + 2xy - x^2}{(x + y)^2}
\]

\[
= \frac{x^2 - 2xy}{(x + y)^2}
\]

\[
\frac{\partial f}{\partial y} = \frac{x^2 \frac{\partial}{\partial y} (x + y) - (x + y) \frac{\partial}{\partial y} x^2}{(x + y)^2}
\]

\[
= \frac{x^2 (-1)(x + y)^2 \frac{\partial}{\partial y} (x + y) - (x + y) \frac{\partial}{\partial y} x^2}{(x + y)^2}
\]

\[
= \frac{-x^2}{(x + y)^2}
\]
2) (25 points) Let \( f(x, y) = 1 - x^2 - y^2 \), \( P = P(0, \frac{1}{2}) \).

a) Sketch the surface; use about a third to the page. Locate \( P, \ f(P) \).

b) In the \( xz \) plane, sketch the trace \( z = f(x, \frac{1}{2}) \). Locate \( P, \ f(P) \)

c) On the surface, sketch the trace \( z = f(x, \frac{1}{2}) \).

\[ f(P) = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{3}{4} \]

\[ f = 1 - x^2 - \frac{1}{4} = \frac{3}{4} - x^2 \]
3) (25 points) Let \( f = f(x, y, z); x = r \cos \theta \sin \phi, y = r \sin \theta \sin \phi, x = r \cos \phi. \)

a) Draw the tree for \( f. \)

b) State the chain rule for \( \frac{\partial f}{\partial \theta}. \)

c) Compute \( \frac{\partial f}{\partial \theta}. \)

d) Let \( f(x, y, z) = x^2 + y^2. \) Use c) to show \( f \) has no \( \theta \)'s in it.

\[
5) \quad \frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \theta}
\]

\[
c) \quad \frac{\partial f}{\partial x} = \frac{\partial f}{\partial \theta} = -y \frac{\partial f}{\partial x} + x \frac{\partial f}{\partial y}
\]

\[
d) \quad \frac{\partial f}{\partial \theta} = 2x \quad \frac{\partial f}{\partial \phi} = 2y
\]

\[
\frac{\partial f}{\partial \theta} = -y(2x) + x(2y) = 0
\]
4)(30 points) Let \( R \) be the region \( R = \{(x,y) \mid x^2 + y^2 \leq 1; \ x \geq 0; \ y \leq 0\} \).

a) Sketch the region \( R \) (use about an eighth of the page).

b) Write the region as a type two region.

c) Compute the integral

\[
\iint_R \sqrt{x^2 + y^2} \, dA
\]

\[
R = \{(x,y) \mid -1 \leq y \leq 0; \ 0 \leq x \leq \sqrt{1-y^2}\}
\]

c) Polar:

\[
\sqrt{x^2 + y^2} \rightarrow \sqrt{r^2} = r
\]

\[
dx \, dy \rightarrow r \, dr \, d\theta
\]

\[
R \rightarrow \left\{(x,y) \mid 0 \leq r \leq 1; \ 3\pi/2 \leq \theta \leq 2\pi\right\}
\]

\[
= \int_{3\pi/2}^{2\pi} \int_0^1 r^2 \, dr \, d\theta
\]

Inside:

\[
\int_0^1 r^2 \, dr = \left[\frac{r^3}{3}\right]_0^1 = \frac{1}{3}
\]

Outside:

\[
\int_{2\pi}^{3\pi/2} \frac{1}{3} \, d\theta = \frac{1}{3} \left[\theta\right]_{2\pi}^{3\pi/2} = \frac{1}{6} \left[\frac{3\pi}{2} - 2\pi\right]
\]

\[
= \frac{\pi}{6}
\]