1)(20 points) Let \( f(x, y) = y/(x^2 + y^2) \). Compute and simplify:

\[
\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \left( \frac{y}{x^2 + y^2} \right) = \frac{y \frac{\partial}{\partial x} (x^2 + y^2) - (x^2 + y^2) \frac{\partial y}{\partial x}}{(x^2 + y^2)^2} = \frac{y(-2x)}{(x^2 + y^2)^2} = -\frac{2xy}{(x^2 + y^2)^2}
\]

\[
\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \left( \frac{y}{x^2 + y^2} \right) = \frac{1 \cdot (x^2 + y^2) - y \cdot 2y}{(x^2 + y^2)^2} = \frac{x^2 - y^2}{(x^2 + y^2)^2}
\]
2) (25 points) Let \( f(x, y) = \sqrt{1 - x^2 - y^2}, P = P(0, \frac{1}{2}) \).

a) Sketch the surface; use about a third to the page. Locate \( P, f(P) \).

b) In the \( xz \) plane, sketch the trace \( z = f(x, \frac{1}{2}) \). Locate \( P, f(P) \).

c) On the surface, sketch the trace \( z = f(x, \frac{1}{2}) \).

\[ f(0) = \sqrt{1 - 0^2 - \frac{1}{4}} = \frac{\sqrt{3}}{2} \]

\[ z = \sqrt{1 - \frac{1}{4} - y^2} = \sqrt{\frac{3}{4} - y^2} \]
3)(25 points) Let \( z = f(x, y); \ x = u \cosh(v), \ y = u \sinh(v). \)

a) Draw the tree for \( f. \)

b) State the chain rule for \( \frac{\partial f}{\partial v}. \)

c) Compute \( \frac{\partial f}{\partial v}. \)

d) Let \( f(x, y) = x^2 - y^2. \) Use c) to show \( f \) has no \( v \)'s in it.

\[
\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial v}
\]

\[
\frac{\partial x}{\partial v} = u \sinh(v) \\
\frac{\partial y}{\partial v} = u \cosh(v)
\]

\[
\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x} u \sinh(v) + \frac{\partial f}{\partial y} u \cosh(v)
\]

\[
= y \frac{\partial f}{\partial x} + x \frac{\partial f}{\partial y}
\]

\[
\frac{\partial f}{\partial x} = 2x \\
\frac{\partial f}{\partial y} = -2y
\]

\[
\frac{\partial f}{\partial v} = y(2x) + x(-2y) = 0
\]
4) (30 points) Let \( R \) be the region \( R = \{(x, y) | x^2 + y^2 \leq 1; x \leq 0; y \geq 0\} \).

a) Sketch the region \( R \) (use about an eighth of the page).

b) Write the region as a type two region.

c) Compute the integral

\[
\int_R \int \sqrt{x^2 + y^2} \, dA
\]

\[ R = \left\{ (x, y) \mid -1 \leq y \leq 0, -\sqrt{1-y^2} \leq x \leq 0 \right\} \]

\[ \sqrt{x^2 + y^2} = r \]

\[ dxdy \rightarrow r \, dr \, d\theta \]

\[ R \rightarrow \left\{ (r, \theta) \mid 0 \leq r \leq 1, \pi \leq \theta \leq \frac{3\pi}{2} \right\} \]

\[
\int_{\pi}^{\frac{3\pi}{2}} \int_0^1 r^2 \, dr \, d\theta \\
= \int_{\pi}^{\frac{3\pi}{2}} \left[ \frac{r^3}{3} \right]_0^1 \, d\theta \\
= \frac{1}{3} \int_{\pi}^{\frac{3\pi}{2}} \, d\theta \\
= \frac{1}{3} \left[ \frac{3\pi}{2} - \pi \right] = \frac{1}{3} \left[ \frac{\pi}{2} \right] = \frac{\pi}{6}
\]
1) (20 points) Let \( f(x, y) = \frac{x^2}{x + y} \). Compute and simplify:

\[
\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}
\]

\[
\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \left( \frac{x^2}{x + y} \right) = \frac{2x(x+y) - x^2}{(x+y)^2}
\]

\[
= \frac{2x^2 + 2xy - x^2}{(x+y)^2}
\]

\[
= \frac{x^2 - 2xy}{(x+y)^2}
\]

\[
\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \left( \frac{x^2}{x + y} \right) = \frac{x^2(\frac{1}{(x+y)^2})(x+y)^2}{(x+y)^2}
\]

\[
= \frac{x^2}{(x+y)^2}
\]
2) (25 points) Let \( f(x, y) = 1 - x^2 - y^2 \), \( P = P(0, \frac{1}{2}) \).

a) Sketch the surface; use about a third to the page. Locate \( P, f(P) \).

b) In the \( xz \) plane, sketch the trace \( z = f(x, \frac{1}{2}) \). Locate \( P, f(P) \).

c) On the surface, sketch the trace \( z = f(x, \frac{1}{2}) \).

\[
\begin{align*}
\text{a)} & \quad f(P) = \sqrt{1 - \left(\frac{1}{2}\right)^2 - \frac{1}{4}} = \frac{3}{4} \\
\text{b)} & \quad f = 1 - x^2 - \frac{1}{4} = \frac{3}{4} - x^2
\end{align*}
\]
3) (25 points) Let \( f = f(x, y, z); x = r \cos \theta \sin \phi, y = r \sin \theta \sin \phi, z = r \cos \phi. \)

a) Draw the tree for \( f. \)

b) State the chain rule for \( \frac{\partial f}{\partial \theta}. \)

c) Compute \( \frac{\partial f}{\partial \theta}. \)

\[
\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \theta} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial \theta}
\]

d) Let \( f(x, y, z) = x^2 + y^2. \) Use c) to show \( f \) has no \( \theta \)'s in it.

\[
\frac{\partial f}{\partial \theta} = -y \frac{\partial f}{\partial x} + x \frac{\partial f}{\partial y}
\]

\[
\frac{\partial f}{\partial \theta} = -y (2x) + x (2y) = 0
\]
4) (30 points) Let $R$ be the region $R = \{(x, y) \mid x^2 + y^2 \leq 1; \ x \geq 0; \ y \leq 0\}$.
   a) Sketch the region $R$ (use about an eighth of the page).
   b) Write the region as a type two region.
   c) Compute the integral
   \[
   \int_R \int \sqrt{x^2 + y^2} \, dA
   \]

   \[\begin{align*}
   R &= \left\{(x, y) \mid -1 \leq y \leq 0; \ 0 \leq x \leq \sqrt{1-y^2}\right\}
   \\
   \end{align*}\]

   \[\begin{align*}
   &c) \quad \text{polar } \quad \sqrt{x^2+y^2} \rightarrow \sqrt{r^2} = r
   \\
   & \quad \quad \quad d\times d\gamma \rightarrow r \, dr \, d\theta
   \\
   & \quad \text{inside } \quad \int_0^{\sqrt{1/3}} \int_0^{3\pi/2} r^2 \, dr \, d\theta = \left[\frac{r^3}{3}\right]_0^{\sqrt{1/3}} = \frac{1}{2}
   \\
   & \quad \text{outside } \quad \int_{3\pi/2}^{2\pi} \int_0^{\sqrt{2\pi}} r^2 \, dr \, d\theta = \frac{1}{2} \left[\theta\right]_0^{2\pi} = \frac{1}{2} \left(2\pi - \frac{3\pi}{2}\right)
   \\
   & \quad \quad \quad = \frac{1}{2} \left[\frac{\pi}{2}\right]
   \\
   & \quad \quad \quad = \frac{\pi}{6}
   \end{align*}\]