Quiz 9a 10 min

Let \( f(x, y) = \frac{x}{x+y} \). Check whether \( f \) is a solution to the PDE:

\[
x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 0
\]

\[
\frac{\partial f}{\partial x} = \frac{1(x+y) - x(1)}{(x+y)^2} = \frac{y}{(x+y)^2}
\]

\[
\frac{\partial f}{\partial y} = \frac{-x}{(x+y)^2}
\]

\[
x \frac{\partial^2 f}{\partial x^2} + y \frac{\partial^2 f}{\partial y^2} = \frac{x \cdot \frac{y}{(x+y)^2}}{1} + \frac{y(-x)}{(x+y)^2} = 0
\]
Let \( z = f(x, y); \) \( x = uv; \) \( y = u^2 - v^2; \) \( u = t^2, \) \( v = s^2. \)
a) Draw the tree diagram for these variables.
b) State the chain rule for \( \frac{\partial f}{\partial s}. \)
c) Use b) to compute \( \frac{\partial f}{\partial s}. \)

\[
\frac{\partial f}{\partial s} = \frac{\partial f}{\partial u} \frac{\partial u}{\partial s} + \frac{\partial f}{\partial v} \frac{\partial v}{\partial s} \]

Quiz 9c 10 min

Let \( z = f(x, y); \ x = r \cos(\theta + \phi); \ y = r \sin(\theta - \phi). \)

a) Draw the tree diagram for these variables.
b) State the chain rule for \( \frac{\partial f}{\partial r}. \)
c) Use b) to compute \( \frac{\partial f}{\partial r}. \)

\[
\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial r}
\]

\[
\frac{\partial x}{\partial r} = \cos(\theta + \phi) \quad \frac{\partial y}{\partial r} = \sin(\theta - \phi)
\]

\[
\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \cos(\theta + \phi) + \frac{\partial f}{\partial y} \sin(\theta - \phi)
\]