You’re going to compute \( \frac{1}{\sqrt{1+x}} \) using

\[
1 + \left(-\frac{1}{2}\right)x + \left(-\frac{1}{2}\right)\left(-\frac{1}{2} - 1\right)\frac{x^2}{2!} + \left(-\frac{1}{2}\right)\left(-\frac{1}{2} - 1\right)\left(-\frac{1}{2} - 2\right)\frac{x^3}{3!} + \left(-\frac{1}{2}\right)\left(-\frac{1}{2} - 1\right)\left(-\frac{1}{2} - 2\right)\left(-\frac{1}{2} - 3\right)\frac{x^4}{4!} + \cdots
\]

1) Write out \( S_3, \, b_4 \).
2) Simplify the expressions in 1).
3) Use \( S_3 \) to approximate \( \frac{1}{\sqrt{1.2}} \); write as a decimal.
4) How large can \textit{error}_A be? Use scientific notation.
7a Solutions

a) $S_2 = \text{first three terms}$

\[ 1 + \left( -\frac{1}{2} \right)x + \left( \frac{3}{8} \right) \frac{x^2}{2} \]

$S_3 = \text{next term} = \left( -\frac{1}{2} \right) \left( -\frac{1}{2} \right) \left( -\frac{1}{2} \right) \frac{x^3}{3}$

5) $S_3 = 1 - \frac{1}{2} x + \left( -\frac{1}{2} \right) \left( -\frac{3}{8} \right) \frac{x^2}{2} = \frac{1}{2} \left[ 1 - \frac{1}{2} x + \frac{3}{8} x^2 \right]

b) $b_4 = \left( -\frac{1}{2} \right) \left( -\frac{1}{2} \right) \left( -\frac{1}{2} \right) \left( -\frac{3}{8} \right) \frac{x^3}{3} = \left( -\frac{1}{2} \right) \left( \frac{25}{56} \right) \frac{x^3}{23}

= \frac{5}{16} |x^3|

6) $\frac{1}{\sqrt{1.2}} = \frac{1}{\sqrt{1+0.2}} \Rightarrow x = 0.2$

$S_3 = 1 - \frac{1}{2} (-0.2) + \frac{3}{8} \left( -0.2 \right)^2 = 1 - 0.1 + \frac{3}{8} 4 \times 10^{-2}

= 0.9 + 0.015 = 0.915$

4) $\text{ERRO}_{A} \leq b_4 = \frac{5}{16} |x^3| = \left( \frac{5}{16} \right) \left( 0.2 \right)^3 = \frac{5}{16} \times 10^{-3} = \frac{5}{2} \times 10^{-3}

= 2.5 \times 10^{-3} \text{ down for rounding up}$

\[ \text{ERRO}_A \leq 2.5 \times 10^{-3} \]
You’re going to compute $\frac{1}{\sqrt{1 + x}}$ using

$$1 + \left(-\frac{1}{3}\right)x + \left(-\frac{1}{3}\right)\left(-\frac{1}{3} - 1\right)\frac{x^2}{2!} + \left(-\frac{1}{3}\right)\left(-\frac{1}{3} - 1\right)\left(-\frac{1}{3} - 2\right)\frac{x^3}{3!} + \left(-\frac{1}{3}\right)\left(-\frac{1}{3} - 1\right)\left(-\frac{1}{3} - 2\right)\left(-\frac{1}{3} - 3\right)\frac{x^4}{4!} + \cdots$$

1) Write out $S_3$, $b_4$.
2) Simplify the expressions in 1).
3) Use $S_3$ to approximate $\frac{1}{\sqrt{1.3}}$; write as a decimal.
4) How large can $error_A$ be? Use scientific notation.
8.76 Solutions

a) \[ S_3 = \text{first three terms} \]
\[ = 1 + (-\frac{1}{3})x + (-\frac{1}{3} - \frac{1}{3}) \frac{x^2}{2} \]
\[ = 1 + \frac{1}{3} - \frac{1}{3} \cdot \frac{x^2}{2} \]

b) \[ b_4 = \text{next term do} \]
\[ = (\frac{1}{3}) ( \frac{4}{3} - 2) \frac{x^3}{2!} \]
\[ = \frac{4}{1 \cdot 3} \frac{x^3}{2!} \]

\[ S_3 = 1 \cdot \frac{4}{1 \cdot 3} \left( \frac{x^3}{2!} \right) \]
\[ = \frac{4}{1 \cdot 3} \left( \frac{1 \cdot 1 \cdot 3}{1 \cdot 3} \right) \left( \frac{x^3}{2!} \right) \]
\[ = \frac{4}{3} \frac{x^3}{2!} \]

\[ c) \quad \frac{1}{\sqrt{1 - 0.3}} = \frac{1}{\sqrt{1 - x}} \]
\[ x = 0.3 \]

\[ S_3 = 1 - \frac{1}{3} (0.3) + \frac{2}{9} (0.3)^2 = 1 - 0.1 + \frac{2}{9} \cdot 0.1 \]
\[ = 0.9 + 0.02 = \]
\[ \sqrt{0.92} \]

\[ c) \quad \text{Error} \] in \[ A = \frac{14}{81} \left( 0.3 \right)^3 \]
\[ = \frac{14}{81} \cdot 0.027 \]
\[ = 4.67 \times 10^{-3} \]

\[ \text{Infinite decimal, round up} \]
\[ \text{Error} \leq 4.67 \times 10^{-3} \]
87 a and b Issues

1) you were asked to simplify 55, 64
   \( \frac{14}{81} \) is simplified, \( \frac{210}{215} \) is not
   \( \sqrt{9.15} \)
   If you can't simplify, you can't get the correct answer
   by taking the absolute value of next term along

2) you were asked to produce a number in
   decimal form, fractions not allowed.
   This was worth 40 points. The number
   we like is 92 or 91.5 - these are not
difficult.

3) you needed to estimate error in scientific notation
   your answers should be like
   \( \pm \mathbf{1.67 \times 10^{-3}} \)
   leaving out the costs points, leaving it fractional
   costs 6 points.
   and getting the wrong error costs all 30 points.

Look at the solutions - these things are very easy.
You were asked to produce correct numbers,
so if your numbers were wrong, you lost
a lot of points.