One For You: Traces

1) Let $f(x, y) = x^2 + y^2$. Draw the surface and, on the surface, draw the trace $z = f(\frac{1}{2}, y)$.

2) Let $f(x, y) = \sqrt{1 - (x^2 + y^2)}$. Draw the surface and, on the surface, draw the trace $z = f(x, \frac{1}{2})$. 
1) \( Z = f(x, y) = x^2 + y^2 \). Sketch the surface, and on the surface, sketch the trace \( Z = F(\frac{1}{2}, y) \).

First, let's see what we've got: \( Z = f(\frac{1}{2}, y) = (\frac{1}{2})^2 + y^2 = \frac{1}{4} + y^2 \). This is a parabola.

So, on the trace, \( z = 1 \) draw \( z = \frac{1}{4} + y^2 \).

And since \( x = \frac{1}{2} \) is constant, the graph will only be in \( y \), which means it will look parallel to the \( y \)-axis.

To help us transfer the parabola into 3D, we'll rely on three points, \( P, Q, R \):

\[ \begin{align*}
  P & : (-\infty, -\infty, 0) \\
  Q & : \left(\frac{1}{2}, 0, 0\right) \\
  R & : \left(\frac{1}{2}, 0, \frac{1}{4}\right)
\end{align*} \]

\( R \) is where the parabola \( Z = \frac{1}{4} + y^2 \) has a minimum. That is, at \( y = 0 \), and then \( Z = \frac{1}{4} + 0^2 = \frac{1}{4} \). Since we're drawing the trace \( x = \frac{1}{2} \), \( R = \left(\frac{1}{2}, 0, \frac{1}{4}\right) \).

\( P \) and \( Q \) are where \( z = 1 \) hits \( Z = \frac{1}{4} + y^2 \) so \( z = \frac{1}{4} + y^2 \)

\[ \begin{align*}
  3y & = y^2 \\
  y^2 - 3y & = 0 \\
  y(y - 3) & = 0 \\
  y & = 0, 3
\end{align*} \]

\( P = P\left(\frac{1}{2}, -\frac{3}{2}, 1\right) \) \( Q = Q\left(\frac{1}{2}, \frac{3}{2}, 1\right) \).
Let's start by transforming $R$ to the graph.

To move $R$ to the graph, move up the $y$-axis, from $x = \frac{1}{2}, y = 0$, until you hit the graph.

The $z$-coordinate is $R \rightarrow R \left( \frac{1}{2}, 0, \frac{1}{4} \right)$, so I should look about a quarter of the way up. Actually, it looks about $1/2$.

Now let's translate $P$ and $Q$. They lie on the line $y = \frac{1}{2}$, at $y = \pm \sqrt{3}/2$, $z = 1$. So they look roughly like

The trick is to place them above the line $x = \frac{1}{2}$.
One way to do that is to lift the line \( x = \frac{1}{2} \) up to \( z = 1 \), and see where it hits the circle at \( z = 1 \).

To do that, draw a fake \( x\)-axis at height \( z = 1 \).

Make it parallel to the red \( x\)-axis.

Now go out \( \frac{1}{2} \) unit and draw the dotted line parallel to the \( y\)-axis.

The intersection of the dotted line with the circle gives \( p, q \).

Hopefully, \( q \) looks about \( \sqrt{3/2} \) out along \( y \) — here it doesn't.

Now put it all together on a big graph.
now connect all three

trace

\[ z = f \left( \frac{1}{2}, y \right) \]
2) Let \( z = \sqrt{1-x^2-y^2} \). Draw, on the surface, the trace \( z = f(x, \frac{1}{2}) \).

First, see what this looks like in the \( xy \) plane:

\[
3 = \sqrt{1-x^2-(\frac{1}{2})^2} = \sqrt{1-\frac{1}{4}} - x^2 = \sqrt{\frac{3}{4} - x^2}
\]

This is a circle of radius \( \frac{\sqrt{3}}{2} \).

Again, \( \phi \) is easy — it occurs at maximum value — that is, \( x = 0 \).

So \( \phi = \phi(0, \frac{1}{2}, \frac{\sqrt{3}}{2}) \).

But \( \theta, \phi \) are easy too—they are also at ends of the radius of circle, so they are \( \pm \frac{\sqrt{3}}{2} \).

Since \( \phi = 0 \), \( P = P(-\frac{\sqrt{3}}{2}, \frac{1}{2}, 0) \) and \( R = R(\frac{\sqrt{3}}{2}, \frac{1}{2}, 0) \).

Now transfer to the surface.
Now just draw the curve on the surface so it connects P, Q, R and looks like a half-circle.

Not very circle-ish.