YAPS: Yet Another Problem Set  \( u \)-Substitution

The following are questions/issues to help you to organize your thoughts. The answers are to be found by looking at the work we did in class.

Section:  \( u \)-Substitution

1) One chain rule is power-chain; an example would be \( [(\sin x)^2]' = 2(\sin x)(\sin x)' \).
   a) Give examples of integrals that use a \( u \)-sub power-chain type of chain rule.
   b) What in these example tell you that a power-chain \( u \)-substitution is the right thing to do?
   c) How would you describe, in words, what \( u \) to choose?

2) The general chain rule shows how to differentiate any composition. An example would be \( \sin(x^2)' = \cos(x^2)(x^2)' \).
   a) Give examples of integrals that use the general \( u \)-sub type of chain rule.
   b) Repeat as in 1)

3) If you’re using a \( u \)-sub in a definite integral, you have to change the limits of integration.
   a) Give an example of doing this.
   b) In the integral \( \int_0^1 \frac{x}{\sqrt{1-x}} \, dx \), the \( u \)-sub changes the limits to \( \int_1^0 \). How do you change them back to the 'right' order (smaller number as the bottom limit of integration)?
   c) The problem in 3b above is an example of a 'left behind' integral. How do you treat these?

Section: Other Integration

1) We’ve lightly covered inverse trig integral.
   a) How would you treat an integral like \( \int \frac{1}{2+x^2} \, dx \)?
   b) If you got a definite integral, you might have to compute quantities like \( \tan^{-1}(1) \) or \( \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) \). How would you go about doing those computations?

2) We discussed 'cancellation and re-inforcement'.
   a) How do you treat even and odd functions when you have a definite integral?
   b) Which of these are integrals where you could use even/odd ideas? \( \int_{-1}^{2} x^2 + x^3 \, dx \);
      \( \int_{-1}^{1} x\sqrt{1-x} \, dx \)?

3) What does it mean that "sums in numerators split"? Give an example.