Section 1.x: Division By Zero

When you start talking about these things, it’s easy to get into long pointless arguments: ‘it should be this.”No, it has to be that.’ Like asking ‘did he dope or not?’ You can go around forever.

In society, we resolve those kinds of disputes with laws and trials and arbitration boards. These are supposed to set the societal standards where we appeal for justice. In math, we do it differently: the laws and standards are definitions and theorems.

Definition

\[ \frac{a}{b} = c \] means \[ a = bc \]

Now try saying something like \( \frac{1}{0} = 2 \). Math says that what you must really mean by that is \( 1 = 2 \cdot 0 \). Or, you’re saying \( 1 = 0 \).

This what a definition does for you: it clears away a lot of words and gives you something you can check, something you can immediately see is right or wrong. And you see right away that \( 1/0 = x \) is never gonna work, no matter what \( x \) we try. We say: \( 1/0 \) does not exist. And we mean there’s no \( x \) that exists which would be equal to \( 1/0 \).

Now let’s try it with \( 0/0 \). You could say, ‘any number divided by itself is one’ or you could say ‘\( 0/0 = 0 \cdot (1/0) \)’ and any number times zero is zero.’ You can say lots of things, but how does it check with the definition?

\[ \frac{0}{0} = 1 \] means \( 0 = 0 \cdot 1 \) true!
\[ \frac{0}{0} = 0 \] means \( 0 = 0 \cdot 0 \) true!
\[ \frac{0}{0} = 17 \] means \( 0 = 0 \cdot 17 \) true!

They’re all true! That can’t be right. So we have something different: for \( 1/0 \) we couldn’t find anything it was equal to; for \( 0/0 \) it can be equal to anything. Hm....

This time we’ll say: \( 0/0 \) is indeterminate.