

6. If you were to place a mirror at the top of this page, the digits 0, 1, and 8 would appear the same in the mirror as they do on the page. All the other digits – 2, 3, 4, 5, 6, 7, and 9 – would look different. Any number that uses only these digits, such as 18 or 801, for example, would also appear the same in the mirror as they do on the page. Let's call such numbers *mirror numbers*. For each of the following questions about mirror numbers, provide an explanation along with your answer. (Hint: what does this have to do with the Alphabitia exploration?)

- (a) What is the next mirror number after 18?
- (b) What is the last mirror number before 800?
- (c) What is the 100th mirror number?

This problem synthesizes a few of the concepts we've been discussing in class — problem solving, pattern recognition, and counting in different bases. One helpful way of looking at this problem is to recognize that the mirror numbers form a base 3 enumerative system (with 8 being the symbol for ~~two~~ ^{two}). Thus, for example, we know that 18 is the 5th mirror number, because it has a 1 in the threes place and an 8 in the ones place, meaning "one three and two ones", or 5.

a) 18 is one three and two ones. If we add another one, we get one three and three ones. Replace the three ones with one three and we are left with two threes. Now we just have to write the mirror number that represents two threes. This is 80.

b) 800 is two nines. If we take away one, we are left with one nine and eight ones. Regrouping the eight ones into threes and ones, we may rewrite this as one nine, two threes, and two ones. The mirror number that represents this quantity is 188.

(Do you see the similarities with base 10 counting here?)

c) There is a reason I asked for the 100th mirror number, rather than the 4th mirror number — I didn't want you to try counting that high. In order to do this problem, you need to write 100 in base 3. Notice that $100 = 1(3^4) + 0(3^3) + 2(3^2) + 0(3^1) + 1(3^0)$. In other words, one hundred is one eighty-one, no twenty-sevens, two nines, no threes, and one one. In mirror numbers, this is 10801. (I also gave credit for 10800, which is what you would get if you counted zero as the first mirror number.)