

M316 Final Exam

December 1, 2008

Name _____

This is a closed book, closed note exam. It consists of 8 problems, plus 3 extra credit problems. Each problem is worth 12 points, and each extra credit problem is worth 3 points. In addition, you will receive 4 points for writing your name. You may not use a calculator on this exam. Good luck!

1. Alexandra's used car lot is offering an additional 15% off cars that are already 20% off the original price. Meanwhile, Mira's used car lot is advertising 35% off the original price for all the cars on the lot. If the car you want is available at both places (at the same original price), where should you go and why? How much will you save by going to the one used car lot as opposed to the other?

2. (a) Which of the numbers below are natural numbers? Which are integers? Rational numbers? Real numbers? Fill in the chart below by writing “yes” if the number belongs to the corresponding set, and “no” if not. (Note: a number may belong to more than one of these sets.)

	N	Z	Q	R
$\frac{3}{4}$				
1				
-6.71				
$\pi + 2$				

- (b) What is the additive inverse of -5? Why?
- (c) There is one rational number that does not have a multiplicative inverse. What is it? Why doesn't this number have a multiplicative inverse?
- (d) Write a word problem that corresponds to this arithmetic problem:
 $(-6) \cdot (-\frac{2}{3})$.

3. (a) Write the number 81 in base 6.

(b) Write the number 103_5 in base 10 (recall that the subscript 5 indicates that the number is currently written in base 5).

(c) Do the following base 5 arithmetic problem: $30001242_5 \times 21_5$. (You may leave your answer in base 5.)

4. (a) Peter is just beginning to learn to count using our base 10 enumerative system. He doesn't understand why 30 is the next number after 29. How would you explain it to him?
- (b) Which of the four fundamental operations of arithmetic satisfy the commutative property?
- (c) What is $200,000 + 600,000$?
- (d) What is $21+43$?
- (e) Recall the distributive property of multiplication: if a , b , and c are numbers, then $a(b + c) = ab + ac$. In your solutions to problems (c) and (d) above, you implicitly used this property. Explain how.

5. (a) Determine the prime factorization of 231.
- (b) What is the smallest number with 7 factors? What is the smallest number with 6 factors?
- (c) Haley's comet can be seen every 75 years. The Swift-Tuttle comet can be seen every 135 years. If one year we see them both at the same time, how long will it be before they are both visible in the same year again?

6. For each of the four problems below, select the model from the list that fits. After briefly justifying your choice, solve the problem.

Models:

- + Combine, increase
- Take-away, comparison, missing addend
- × Repeated addition, area, Cartesian product
- ÷ Partitioning, repeated subtraction, missing factor

- (a) Julia is preparing a meal. She knows how to cook 13 different entrees and 17 different side dishes. If a meal is a combination consisting of one entree and one side dish, how many different possible meals can Julia cook?

- (b) In Portland, Maine, it was -11 degrees Fahrenheit on December 21st. After 8 days, the temperature warmed up to -5 degrees Fahrenheit. How much warmer was it in Portland on December 29th than it was on December 21st?

(c) My car has an average fuel efficiency of 43 miles per gallon. If I can travel 417.1 miles on a tank of gas, how large (in gallons) is my gas tank?

(d) One day, Elizabeth decides to walk home from the office. If the total distance from her office to her home is $2\frac{1}{4}$ miles, and she has walked $\frac{2}{3}$ of the way, how far away from the office is she?

7. (a) On Josh's birthday, his mom cuts his birthday cake into 6 pieces and gives him 1. On Sammy's birthday, his mom cuts his cake into 12 pieces and gives him 2. Josh begins to cry because Sammy got more pieces of cake on his birthday than Josh got on his. How would you explain to Josh that he actually got the same amount of cake as Sammy?

(b) On Jerry's birthday, his mom cuts his birthday cake into 20 pieces and gives him 3. Now Sammy is crying because Jerry got more pieces of birthday cake than he did. How would you explain to Sammy that he actually got more cake than Jerry did?

- (c) Now that Sammy knows that he got more cake than Jerry, he wants to know how much more. Determine how much more cake Sammy got than Jerry, and then describe how you would explain your solution to Sammy.

8. Let a, b be two numbers such that $0 < b < a < 1$. In the table below, fill in the second column with either $<$, $=$ or $>$ so that the first three columns give a true number sentence. In the fourth column, give a justification for your choice of $<$, $=$, or $>$. (Note: examples do not constitute sufficient justification and will not receive full credit.)

$a \times b$		a	
$a + b$		b	
$a \div b$		a	
$a - b$		a	

Extra Credit: What is your instructor's favorite number?

Extra Credit: What is the last digit of 9^{100} ?

Extra Credit: Write the following rational number in the form $\frac{a}{b}$, where a and b are integers: $0.027027027027027\dots$