M361 Assignment 4

Due in class Thursday, September 25.

1. Evaluate
   \[ \lim_{z \to i} \frac{z^2 + iz + 2}{z^2 + 1} \]
   (You may use any results discussed in lecture, i.e. you do not need to use the \( \epsilon-\delta \) definition).

2. (a) Find functions \( f, g : \mathbb{C}\{0\} \to \mathbb{C} \) such that \( \lim_{z \to 0} (f(z) + g(z)) \) exists but \( \lim_{z \to 0} f(z) \) does not exist.
   (b) Find functions \( f, g : \mathbb{C}\{0\} \to \mathbb{C} \) such that \( \lim_{z \to 0} (f(z)g(z)) \) exists but \( \lim_{z \to 0} f(z) \) does not exist.

3. Give an example of a continuous function \( f : S \to \mathbb{C} \) such that \( S \) bounded but \( f \) is unbounded.

4. (a) Show that
   \[ \lim_{w \to 0} \frac{\overline{w}^2}{w} = 0. \]
   \textit{Hint: You may use the fact that } \( \lim_{z \to z_0} f(z) = 0 \text{ if and only if } \lim_{z \to z_0} |f(z)| = 0. \)
   (b) Using part (a), show that the origin is the only point where the function \( f : \mathbb{C} \to \mathbb{C}, f(z) = \overline{z}^2 \) is differentiable.

Exercises from the textbook: