M361 Assignment 2

Due in class Thursday, September 11.

1. Recall that for $z = x + iy$ we make the definition

$$e^z = e^x e^{iy} = e^x(\cos y + i \sin y).$$

   Prove that $e^{z_1+z_2} = e^{z_1}e^{z_2}$ for all $z_1, z_2 \in \mathbb{C}$ (you may use anything proven in lecture).

2. Prove that $e^{z_1} = e^{z_2}$ if and only if $\operatorname{Re} z_1 = \operatorname{Re} z_2$ and $\operatorname{Im} z_1 = \operatorname{Im} z_2 + 2\pi k$ for some $k \in \mathbb{Z}$.

3. Define the following functions from $\mathbb{C} \to \mathbb{C}$

$$C(z) = \frac{e^{iz} + e^{-iz}}{2}$$
$$S(z) = \frac{e^{iz} - e^{-iz}}{2i}.$$

   (a) Show that $e^{iz} = C(z) + iS(z)$.

   (b) Show that $C(z)^2 + S(z)^2 = 1$.

   (c) Show that $C(z) = C(z + 2\pi k)$ and $S(z) = S(z + 2\pi k)$ for $k \in \mathbb{Z}$.

   (d) Show that if $z = x$ is real then $C(x) = \cos x$ and $S(x) = \sin x$.

   (e) Show that $C(z)$ and $S(z)$ are each unbounded. That is, show that for any $R \in \mathbb{R}, R > 0$ there exists some z such that $|C(z)| \geq R$ (similarly for $S(z)$).

   Remark: We will see that C and S are the natural extensions of the functions cos *and* sin *to the complex plane. Note the familiarity of properties (a) – (d) but the unfamiliarity of property (e).*

Exercises from the textbook:
p. 23: #9.
p. 29-30: #1(a)(b), #3(a) (ignore "identify the principal part"), #7.