

**SOME TRIG STUDY PROBLEMS FOR THE FINAL**  
**M305G, FALL 2008**

Below I have provided some problems which may be helpful in studying for the final. These problems are primarily based on material from the last 2.5 weeks of the course. While these may serve as good practice problems they **do NOT comprise a complete study guide for the final exam**. The final exam is a cumulative test. This means that all the material covered in this class is fair game. Some good **study guides are quizzes, midterm study guides, midterm exams, homework, lecture notes and your book**. Comments and solutions for quizzes, homework and midterm exams are all posted on the course website. The better you understand these comments and solutions the more likely you are to not repeat previous mistakes on the final.

- (1) What is the domain of  $\sqrt{\frac{x-4}{(x+2)(x-3)}}$
- (2) What is the domain of  $\log_{43}(x-4)$
- (3) What is the domain of  $\ln\left(\frac{x+3}{(x-1)(x+2)}\right)$
- (4) Use the trigonometric sum formulas to show that  $\sin\left(\theta + \frac{\pi}{2}\right) = \cos(\theta)$ .
- (5) Use the trigonometric sum formulas to show that the function  $f(\theta) = \cos(\theta)$  has period  $2\pi$ .
- (6) Find the exact value of  $\cos\left(\frac{\pi}{12}\right)$
- (7) Find the exact value of  $\sin\left(\frac{7\pi}{12}\right)$
- (8) Find the exact value of  $\sin\left(\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{-3}{5}\right)\right)$
- (9) Find the exact value of  $\cos\left(\tan^{-1}(1/2) - \sin^{-1}(1/2)\right)$
- (10) Consider the function  $y = -4\sin(2\pi x) + 1$ 
  - What's the period?
  - What's the range?
  - What's the amplitude?
  - Draw and label the graph.
- (11) Consider the function  $f(x) = \frac{1}{2}\cos\left(\frac{3}{\pi}x\right) - \frac{1}{2}$ 
  - What's the period?
  - What's the range?
  - What's the amplitude?
  - Draw and label the graph on the interval  $[-\pi/2, 3\pi]$
  - On the interval  $[-\pi/2, 3\pi]$ , where is  $f(x)$  increasing? Where is  $f(x)$  decreasing? Where does  $f(x)$  have a local minimum? Where does  $f(x)$  have a local maximum?
- (12) Consider the function  $g(x) = 3\tan(4x)$ 
  - What's the period?
  - What's the range?
  - What's the amplitude?
  - Draw and label the graph on the interval  $[-\pi/2, \pi/2]$

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- On the interval  $[-\pi/2, 3\pi]$ , where is  $f(x)$  increasing? Where is  $g(x)$  decreasing? Where does  $g(x)$  have a local minimum? Where does  $g(x)$  have a local maximum?
- (13) Consider the function  $y = \sin(\pi x - \frac{\pi}{3})$ 
    - What's the period?
    - What's the range?
    - What's the amplitude?
    - Draw and label the graph.
  - (14) Write an equation for a sine function with amplitude 2 and period  $\frac{\pi}{2}$
  - (15) Write an equation for a cosine function with range  $[-2, 4]$ .
  - (16) Does it make any sense to talk about the amplitude of a tangent function? Explain.
  - (17) Find the exact value of  $\cos(\tan^{-1}(1/3))$
  - (18) Find the exact value of  $\tan(\cos^{-1}(-2/5))$
  - (19) Find the exact value of  $\sin^{-1}(\sin(\frac{5\pi}{8}))$
  - (20) Find the exact value of  $\cos^{-1}(\cos(\frac{5\pi}{3}))$
  - (21) For what values of  $x$  does  $\cos^{-1}(\cos(x)) = x$ ?
  - (22) For what values of  $x$  does  $\sin(\sin^{-1}(x)) = x$ ?
  - (23) For what values of  $x$  does  $\tan^{-1}(\tan(x)) = x$ ?
  - (24) For what values of  $x$  does  $\tan(\tan^{-1}(x)) = x$ ?
  - (25) Find the exact value of  $\sec^{-1}(2)$
  - (26) Find the exact value of  $\csc^{-1}(\frac{-2}{\sqrt{3}})$
  - (27) Establish the identity,  $2\ln|\tan(\theta)| = \ln|\sec(\theta) + 1| + \ln|\sec(\theta) - 1|$
  - (28) Establish the identity,  $(4\sin(x)\cos(x))(1 - 2\sin^2(x)) = \sin(4x)$
  - (29) Find  $\tan(4\theta)$ ,  $\sin(2\theta)$ , and  $\cos(\frac{\theta}{2})$  given that  $\sin(\theta) = -2/3$  and  $\pi \leq \theta \leq \frac{3\pi}{2}$ .
  - (30) Section 7.5: # 9,11,12, 30, 32,33,34, 68, 77,78
  - (31) Section 7.4: # 14,15,17,17,32, 34, 40,44, 78, 80
  - (32) Section 7.4: # 38
  - (33) What does it mean for a function to be periodic?
  - (34) What does it mean for a function to be one-to-one.
  - (35) Is a periodic function ever one-to-one? If so give an example. If not, give an explanation.
  - (36) If the functions  $f(x)$  and  $g(x)$  are inverses of each other what can you say about  $f \circ g(x)$  and  $g \circ f(x)$ ?
  - (37) What is the definition of a radian.
  - (38) Use the unit circle to define the functions  $\sin(\theta)$  and  $\cos(\theta)$ .
  - (39) Explain how to find the angle of inclination of the sun using a rod and its shadow.
  - (40) Explain why a function must be one-to-one in order to have an inverse.
  - (41) Give the definition of the **graph** of a function.
  - (42) Give the definition of the **domain** of a function.
  - (43) Solve the equation  $36^x + 3(6^x) + 2 = 0$  for  $x$ .
  - (44) Find an exact solution for  $x$  if  $32(8^x) = 4$ .
  - (45) Solve the equation  $3^{2x-1} = 7^{x+1}$  for  $x$ .
  - (46) Solve the equation  $\log(x) + \log(x - 4) = \log(x - 6)$  for  $x$ .
  - (47) Solve the equation  $2\log_{31}(x) = \log_{31}(121)$  for  $x$ .

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- (48) Draw the unit circle. Label the angles  $\alpha = \frac{\pi}{3}$  and  $\beta = \frac{5\pi}{4}$ . Label the coordinates of their points of intersection with the unit circle as well as  $\sin(\alpha)$ ,  $\sin(\beta)$ ,  $\cos(\alpha)$ , and  $\cos(\beta)$ .

Let  $P_\alpha$  and  $P_\beta$  be the points of intersection of the angles  $\alpha$  and  $\beta$  with the unit circle.

- Find the distance of the line segment between  $P_\alpha$  and  $P_\beta$  .
- Find the coordinate of the midpoint the line segment between  $P_\alpha$  and  $P_\beta$  .
- Find the distance between the midpoint of the line segment and the end point  $P_\alpha$
- Write the equation of the line through the points  $P_\alpha$  and  $P_\beta$  .