ANOTHER PRACTICE FINAL

1. Evaluate the following limits and improper integrals or write "DNE" if the limit does not exist. EXPLAIN YOUR REASONING
   a. $\int_1^\infty \frac{100x^2}{3x^5+2}$
   b. $\int_0^3 \frac{1}{(e^x-1)^3}$
   c. $\lim_{x\to 3} x^{\frac{1}{x^2}}$
   d. $\lim_{x\to 4} \frac{ln(x^4)-4ln(4)}{x^3-64}$
   e. $\lim_{x\to\infty} \frac{arctan(x)}{x^5}$

2. Indicate which of these series converge absolutely, which converge conditionally, and which diverge. EXPLAIN YOUR REASONING.
   a. $\Sigma_{n=1}^\infty \frac{(-1)^n ln(n)}{n}$
   b. $\Sigma_{n=2}^\infty \frac{(-1)^n}{n^2 ln(n)}$
   c. $\Sigma_{n=2}^\infty \frac{sin(n)}{n^2}$
   d. $\Sigma_{n=1}^\infty arctan(n)^{n/2}$
   e. $\Sigma_{n=1}^\infty \frac{32^n n! ln(n)}{n!}$
   f. $\Sigma_{n=1}^\infty \frac{100x^2}{3x^5+2}$

3 Consider the series $f(x) = \Sigma_{n=1}^\infty \frac{x^n}{n^3}$
   a. Find the radius of convergence of $f(x)$.
   b. Use the series to estimate $f(0.3)$. Your answer should be good to 3 decimal places.
   c. Estimate $\int_0^{0.1} f(x)dx$ to 3 decimal places.
   d. Estimate $f'(0.1)$ to 3 decimal places.

4. Let L be the line parallel to the line L_1 defined by $< x, y, z >=< 1, 1, 1 > +t < 1, 2, 1 >$ through the point $(-1, 2, 1)$. Let P be the plane contaning L and L_1. Let Q be the point $Q = (2, 1, -1)$.
   a. What is the equation of the line L?
   b. Find the equation of the plane P.

c. Find the distance from Q to L.

d. Find the distance from Q to P.

5. Consider the parametrized curve $r(t) = (5cos(t), 4sin(t), 3sin(t))$.
   a. Find the velocity vector at time $t = 3$.
   b. What is the angle between the velocity and accelaration vectors at time $t = 3$?
   c. Find the speed as a function of time.
   d. Find the distance traveled from time $t = 1$ to $t = 3$.

6. a. Suppose the angle of a sector is changing at a rate of $\frac{\pi}{6}$ radians per second and the radius is changing at a rate of 6 mm/s. What is the change in area of a sector of radius 3 and angle $\theta = \frac{\pi}{3}$?
   b. A fish is swimming through a bundle of kelp as a speed of 10 m/s in the direction $< 2, 3, -1 >$. The temperature of the water is given by $T(x, y, z) = xyz^2 + 1$.
      i. At what rate is the temperature changing as the fish swims through the point $< -1, \frac{1}{2}, 1 >$?
      ii. From $< -1, \frac{1}{2}, 1 >$, what direction would the fish have to swim so that the rate of chance of temperature was a maximum?

7. Consider the surfaces $S_1 = \{(x, y, z)|z^2 + y^2 + z^2 - 4z\}$ and $S_2 = \{(x, y, z)|z^2 + y^2 + z^2 - 4\}$.
   a. Find the planes tangent to S_1 and S_2 at the point $(\sqrt{3}, 0, 1)$.
   b. Find the lines through $(\sqrt{3}, 0, 1)$ which are parallel to the normals of the planes in part (a).
   c. Find the surface area of the region bounded by the two surfaces S_1 and S_2.
   d. Find the volume of the region bounded by the two surfaces S_1 and S_2.

8. Consider the function $f(x, y) = (e^{x^2} - x^3)(y^2 + y)$.
   a. Find all of the critical points.
   b. Use the second derivative test to determine which critical points are local maxima, which are local minima and which are saddle points.

9. a. Find the area of the triangle with vertices $(0, 0)$, $(3, 0)$, and $(3, 9)$ by

i. using the cross product.
ii. using double integrals and integrating with respect to x first.

b. Find the volume of the surface defined by $10 = x^2 + y^2 + z$ which lies over the region described in part (a).

c. Find the surface are of the paraboloid $10 = x^2 + y^2 + z$ that lies over the region from part (a).

10. a. Find the volume of the solid $z = 8 - x^2 - y^2$ bounded by the plane $z = 3$.

b. Find the surface area of the solid $z = 8 - x^2 - y^2$ inside the cylinder $x^2 + y^2 = 4y$.

11. A lamina takes up the region $R = \{(x, y) | 1 \leq x^2 + y^2 \leq 4, y \leq x\}$. The mass desity of the lamina is $\rho(x, y) = e^{x^2+y^2} \frac{x}{\sqrt{x^2+y^2}}$.

a. Find the total mass of the lamina.

b. Set up but do not evaluate the integrals that define the center of mass $(\bar{x}, \bar{y})$.

12. What is $\int_0^\infty e^{-8x^2} dx$?