09/07/17

Last Time: Inverse Problems
Logarithms
Velocity (average)

Today: Limits

Future: HW00 Due tomorrow
HW02 Due Monday

Ex: Let \( s(t) = 3t^2 - 10t + 4 \), where \( s(t) \) is the position of the object. Find the average velocity from \( t=0 \) to \( t=1 \):

\[ V_{avg} = \frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1} \]

Ex: From \( t=0 \) to \( t=1 \), \( \frac{s(1) - s(0)}{1 - 0} = \frac{-3 - 4}{1} = -7 \)

Ex: From \( t=1 \) to \( t=3 \), \( \frac{s(3) - s(1)}{3 - 1} = \frac{[3] - [3]}{2} = \frac{4}{2} = 2 \)

Ex: From \( t=0 \) to \( t=10 \), \( \frac{s(10) - s(0)}{10 - 0} = \frac{204 - 4}{10} = \frac{200}{10} = 20 \)

The tangent line at \( t=3 \) has slope \( 20 \). This slope is the instantaneous velocity at \( t=3 \).
In the picture, $\lim_{x \to 2} f(x) = 4$.

The limit of $f(x)$ as $x \to a$ is whatever $f(x)$ value we approach as $x$ slides towards $a$.

Ex: $\lim_{x \to 1.5} f(x)$ does not exist.
\[ \lim_{x \to a} f(x) = L \quad \iff \quad \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L \]

Find \( \lim_{x \to 1} \frac{3}{(x-1)^2} \) => what is \( \lim_{x \to 1^-} f(x) = \)

\[ \frac{3}{(1-1)^2} = \frac{3}{(-1)^2} = \frac{3}{.01} = 300 \]

\[ \lim_{x \to 1^-} f(x) = \infty \]

If \( x = .9 \Rightarrow \frac{3}{(.9-1)^2} = \frac{3}{(-.1)^2} = \frac{3}{.01} = 300 \)

\[ \lim_{x \to 1^-} f(x) = \infty \]

\[ \frac{3}{(1.1-1)^2} = \frac{3}{.01} = 300 \]

\[ \lim_{x \to 1^+} f(x) = \infty \]

\[ \lim_{x \to 1^-} f(x) = \infty \] [NOT TRUE].
\[ f(x) = \frac{4x}{x + 2}, \quad \text{find} \quad \lim_{{x \to -2}} f(x) = \]

If we let \( x = -2.1 \):
\[ \frac{4(-2.1)}{-2.1 + 2} = \frac{-8.4}{-0.1} = 84 \]

\[ \therefore \lim_{{x \to -2^-}} f(x) = \infty \]

If we let \( x = -1.9 \):
\[ \frac{4(-1.9)}{-1.9 + 2} = \frac{-7.6}{0.1} = -76 \]

\[ \therefore \lim_{{x \to -2^+}} f(x) = -\infty \]

\[ \therefore \lim_{{x \to -2}} f(x) = \text{D.N.E.} \]

Qn: What if we are given a more difficult function?

\[ \lim_{{x \to 1}} f(x), \quad \text{where} \quad f(x) = \begin{cases} 3x & x < 1 \\ x + 2 & x \geq 1 \end{cases} \]

\[ \lim_{{x \to 1^-}} f(x) = 3 \]
\[ \lim_{{x \to 1^+}} f(x) = 3 \]
\[ \lim_{{x \to 1}} f(x) = 3 \]
Draw a graph with the following properties:

- \( \lim_{{x \to 1}} f(x) = 2 \)
- \( f(1) = 4 \)
- \( \lim_{{x \to 3^-}} f(x) = -1 \)
- \( \lim_{{x \to 3^+}} f(x) = 1 \)
- \( f(3) = 1 \)
- \( \lim_{{x \to 5}} f(x) \) exists
- \( f(5) \) DNE
\[ \lim_{x \to 0} \sin \left( \frac{\pi}{x} \right) = \text{D.N.E. because it oscillates as we approach zero.} \]

\[ \lim_{x \to 0^-} f(x) \text{ or } \lim_{x \to 0^+} \text{ neither exist.} \]