09/05/17

Last Time - u-sub
    FDP (easy)

Today - TBP (hard)
    Trig Integration

Future - HW 01 Due Wed *
    HW 00 Due Fri
    HW 02 Due Mon

\[ \int x^2 \sin(2x) \, dx \]

\[ \int \frac{\ln(x)}{x^2} \, dx \]

\[ \int x^2 \sin(2x) \, dx = -\frac{x^2}{2} \cos(2x) - \int \frac{3}{2} \cos(2x) \, dx \]
\[ \text{let } u = x^2, \quad dv = \sin(2x) \, dx \]
\[ du = 2x \, dx, \quad v = -\frac{1}{2} \cos(2x) \]

\[ \int x^2 \sin(2x) \, dx = \frac{1}{2} x^2 \sin(2x) - \int \frac{3}{2} x^2 \cos(2x) \, dx \]
\[ \text{stop} \]
\[ \int x^2 \sin(2x) \, dx = \frac{-x^2}{2} \cos(2x) + \int x \cos(2x) \, dx \]

\[
 u = x \\
 du = dx \\
 v = \frac{1}{2} \sin(2x) \\
 dv = \cos(2x) \, dx
\]

\[ = \frac{-x^2}{2} \cos(2x) + \frac{x}{2} \sin(2x) - \int \frac{1}{2} \sin(2x) \, dx \]

\[ = \frac{-x^2}{2} \cos(2x) + \frac{x}{2} \sin(2x) + \frac{1}{4} \cos(2x) + C \]

\[ \int e \frac{\ln(x)}{x^2} \, dx = \int x^{-2} \ln(x) \, dx = -\frac{\ln(x)}{x} + \int x^{-2} \, dx \]

\[
 u = \ln(x) \\
 du = \frac{1}{x} \, dx \\
 v = -\frac{1}{x} \\
 dv = x^{-2} \, dx
\]

\[ = -\frac{\ln(x)}{x} - \frac{1}{x} \bigg|_{1}^{e} = \left[ -\frac{\ln(e)}{e} - \frac{1}{e} \right] - \left[ -\frac{\ln(1)}{1} - \frac{1}{1} \right] \]

\[ = \frac{-2}{e} - 1 = \left[ 1 - \frac{2}{e} \right] \]
\[ \int 2e^{\sqrt{x}} \, dx = 2 \int e^{\sqrt{x}} \, dx = 2 \int e^u \cdot 2\sqrt{x} \, du = 4 \int ue^u \, du \]

\[ u = x^{\frac{1}{2}} \]
\[ du = \frac{1}{2} x^{-\frac{1}{2}} \, dx \]
\[ dx = 2\sqrt{x} \, du \]

\[ w = u \]
\[ dw = e^u \, du \]
\[ v = e^u \]

\[ x = 4, \ u = 2 \]
\[ x = 1, \ u = 1 \]

\[ = 4 \left[ e^u - \int e^u \, du \right] = 4ue^u - 4e^u \] \[ = \left[ 8e^2 - 4e^2 \right] - \left[ 4e - 4e \right] \]

\[ = 4e^2 \]

\[ \int x^4 e^x \, dx = \int x^4 e^x \, du = \int ue^u \, du = -ue^u + e^u \]

\[ \left[ e^1 - e^0 \right] - \left[ 0 - e^0 \right] = 1 \]
\[ \int \ln(x) \, dx = x \ln(x) - \int dx = (x \ln(x) - x + C) \]

\( u = \ln(x) \quad dv = dx \)
\( du = \frac{1}{x} \, dx \quad v = x \)

\[ [x \ln(x) - x + C]' = \]
\[ [x]' \ln(x) + x[ \ln(x)'] - [x]' \]
\( [\ln(x)] + 1 - 1 = \ln(x) \)

\[ \int \tan^{-1}(x) \, dx = x \tan^{-1}(x) - \int \frac{x}{1+x^2} \, dx \]

\( u = \tan^{-1}(x) \quad dv = dx \)
\( du = \frac{1}{1+x^2} \, dx \quad v = x \)

\[ = x \tan^{-1}(x) - \frac{1}{2} \int \frac{1}{u} \, du = x \tan^{-1}(x) - \frac{1}{2} \ln |1+x^2| + C \]

\[ x \tan^{-1}(x) - \frac{1}{2} \ln |1+x^2| + C \]

\[ x \tan^{-1}(x) - \frac{1}{2} \ln (1+x^2) + C \]

\[ x \tan^{-1}(x) - \ln \sqrt{1+x^2} + C \]
\[
\int e^x \sin(2x) \, dx = e^x \sin(2x) - 2 \int e^x \cos(2x) \, dx \\
\text{Let } u = \sin(2x) \quad dv = e^x \, dx \\
u' = e^x \quad dv = \cos(2x) \\
du = 2 \cos(2x) \quad v = \frac{1}{2} \sin(2x) \\
\int e^x \sin(2x) \, dx = e^x \sin(2x) - 2 \left[ \frac{1}{2} e^x \sin(2x) - \frac{1}{2} e^x \cos(2x) \right] \\
= \int e^x \sin(2x) \, dx \\
\int e^x \sin(2x) \, dx = e^x \sin(2x) - 2 \int e^x \cos(2x) \, dx \\
\text{Let } u = \cos(2x) \quad dv = e^x \, dx \\
u' = -2 \sin(2x) \quad v = e^x \\
\int e^x \sin(2x) \, dx = e^x \sin(2x) - 2 \left[ e^x \cos(2x) + \frac{1}{2} e^x \sin(2x) \right] \\
= e^x \sin(2x) - 2 e^x \cos(2x) - 4 \int e^x \sin(2x) \, dx \\
\int e^x \sin(2x) \, dx = \frac{e^x \sin(2x) - 2 e^x \cos(2x)}{5} + C
\]
\[
\int_0^{2\pi} e^{2x} \sin(x) \, dx = -\cos(x) e^{2x} + \int_0^{2\pi} 2e^{2x} \cos(x) \, dx
\]

\[
\begin{align*}
\int_0^{2\pi} e^{2x} \sin(x) \, dx &= -e^{2x} \cos(x) + 2e^{2x} \sin(x) - 4 \int_0^{2\pi} e^{2x} \sin(x) \, dx \\
\int_0^{2\pi} e^{2x} \sin(x) \, dx &= \frac{1}{3} \left[ e^{2x} \cos(x) + 2e^{2x} \sin(x) \right]_0^{2\pi} \\
&= \frac{1}{3} \left[ e^{2\pi} \cos(2\pi) + 2e^{2\pi} \sin(2\pi) \right] - \frac{1}{3} \left[ -e^0 \cos(0) + 2e^0 \sin(0) \right] \\
&= \frac{1}{3} \left[ e^{2\pi} + 1 \right]
\]
TRIG INTEGRATION

\[
\sin^2(x) + \cos^2(x) = 1 \quad \Rightarrow \quad \tan^2(x) + 1 = \sec^2(x)
\]

\[
\int \sin^2(x) \cos^3(x) \, dx = \int \cos(x) \cdot \sin^2(x) \cdot \cos^2(x) \, dx
\]

\[
= \int \cos(x) \cdot \sin^2(x) \left[ 1 - \sin^2(x) \right] \, dx
\]

\[
= \int \cos(x) \left[ \sin^2(x) - \sin^4(x) \right] \, dx
\]

Let \( u = \sin(x) \)

\[
du = \cos(x) \, dx
\]

\[
dx = \frac{du}{\cos(x)}
\]

\[
= \int u^2 - u^4 \, du = \frac{1}{3} u^3 - \frac{1}{5} u^5 + C
\]

\[
= \frac{1}{3} \sin^3(x) - \frac{1}{5} \sin^5(x) + C
\]
\begin{align*}
\int_0^{\frac{\pi}{2}} \sin^3(x) \cos^4(x) \, dx \\
= \int_0^{\frac{\pi}{2}} \cos(x) \left[ \sin^3(x) \cdot \cos^4(x) \right] \, dx \\
= \int_0^{\frac{\pi}{2}} \sin(x) \left[ \sin^2(x) \cdot \cos^4(x) \right] \, dx \\
= \int_0^{\frac{\pi}{2}} \sin(x) \left[ \cos^2(x) \cdot \cos^4(x) \right] \, dx \\
= \int_0^{\frac{\pi}{2}} \sin(x) \left[ \cos^6(x) \right] \, dx \\
&= \int_0^{\frac{\pi}{2}} (1 - \cos^2(x)) \cdot \cos^6(x) \, dx \\
&= \int_0^{\frac{\pi}{2}} \cos^6(x) \, dx - \int_0^{\frac{\pi}{2}} \cos^8(x) \, dx \\
&= \int_0^{1} u^6 - u^8 \, du - \int_1^{0} u^6 - u^8 \, du \\
&= \frac{1}{7} u^7 - \frac{1}{9} u^9 \Big|_0^1 = \frac{1}{7} - \frac{1}{9} = \frac{2}{63} - \frac{7}{63} \\
&= \frac{2}{63}
\end{align*}
\[ \int \sin^3(x) \, dx \]
\[ \int \sin(x) \cdot \sin^2(x) \, dx \]
\[ \int \sin(x) \left[ 1 - \cos^2(x) \right] \, dx = -\int 1 - u^2 \, du = \int u^2 - 1 \, du \]
\[ u = \cos(x) \]
\[ du = -\sin(x) \, dx \]
\[ \frac{1}{3} u^3 - u + C \]
\[ = \frac{1}{3} \cos^3(x) - \cos(x) + C \]

\[ \int \sin^3(x) \, dx = \frac{x}{2} - \frac{\sin(2x)}{4} + C \]

\[ \int \frac{1}{2} - \frac{\cos(2x)}{2} \, dx \]
\[ = \frac{x}{2} - \frac{\sin(2x)}{4} + C \]

\[ \int \cos^2(2x) \, dx = \int \frac{1}{2} + \frac{\cos(2x)}{2} \, dx = \frac{x}{2} + \frac{\sin(2x)}{4} + C \]