01/23/2020

Last Time: u-sub

Today: Integration by Parts, §7.1

Trig Integration, §7.2

Future: LAMs

Hw01

Hw00

$$\int_2^5 \frac{x}{x-1}\,dx = \int_1^4 \frac{u+1}{u}\,du = \int_1^4 1+\frac{1}{u}\,du$$

$u = x-1$

$du = dx$

$x = u+1$

$x=2,\ u=1$

$x=5,\ u=4$

$$= u + \ln(u)\Big|_1^4$$

$$= [4+\ln(4)] - [1+\ln(1)]$$

$$= 3 + \ln(4) - \cancel{\ln(1)}^{\,0}$$

$$= 3 + \ln(4) \leftarrow$$

$$= 3 + \ln(2^2)$$

$$= 3 + 2\ln(2) \leftarrow$$

$\int x\sin(x^2)\,dx$, Easy, $u = x^2$, ...

What about $\int x\sin(x)\,dx$

Integration by Parts: $\boxed{\int u\,dv = uv - \int v\,du}$

$$\int x\sin(x)\,dx = -x\cos(x) - \int -\cos(x)\,dx$$

$$= -x\cos(x) + \int \cos(x)\,dx$$

$$= \boxed{-x\cos(x) + \sin(x) + C}$$

$u = x \qquad dv = \sin(x)dx$

$du = dx \qquad v = -\cos(x)$

$$[-x\cos(x) + \sin(x) + C]' = \cancel{-\cos(x)} + x(+\sin(x)) + \cancel{\cos(x)} + 0$$

$$= x\sin(x).$$

$[u \cdot v]' = uv' + vu'$

$\int uv' = \int [u\cdot v]' - \int vu'$

$\int u\,dv = uv - \int v\,du$

$$\int x^2 \cos(2x)\,dx$$

$$= \frac{x^2}{2}\sin(2x) - \int x\sin(2x)\,dx \Rightarrow$$

$u = x^2 \quad dv = \cos(2x)\,dx$

$du = 2x\,dx \quad v = \frac{\sin(2x)}{2}$

$u = \cos(2x) \quad dv = x^2\,dx$

$du = -2\sin(2x)\,dx \quad v = \frac{1}{3}x^3$

$$\Rightarrow \frac{1}{3}x^3\cos(2x) - \int -\frac{2}{3}x^3\sin(2x)\,dx \quad \text{STOP}$$

$$\int x^2\cos(2x)\,dx = \frac{x^2}{2}\sin(2x) - \int x\sin(2x)\,dx$$

$u = x \quad dv = \sin(2x)\,dx$

$du = dx \quad v = \frac{-\cos(2x)}{2}$

$$= \frac{x^2}{2}\sin(2x) - \left[\frac{x}{2}\cos(2x) - \int -\frac{\cos(2x)}{2}\,dx\right]$$

$$= \frac{x^2}{2}\sin(2x) + \frac{x}{2}\cos(2x) - \int \frac{\cos(2x)}{2}\,dx$$

$$= \boxed{\frac{x^2}{2}\sin(2x) + \frac{x}{2}\cos(2x) - \frac{\sin(2x)}{4} + C}$$

$$\int_1^e \frac{\ln(x)}{x^2}\,dx = -\frac{\ln(x)}{x} - \int -\frac{1}{x^2}\,dx = -\frac{\ln(x)}{x} + \int x^{-2}\,dx$$

$$= -\frac{\ln(x)}{x} - \frac{1}{x}\Big|_1^e =$$

$u = \ln(x)$ $dv = x^{-2}\,dx$

$u = \frac{1}{x}\,dx$ $v = -x^{-1} = \frac{-1}{x}$

$$= \left[-\frac{\ln(e)}{e} - \frac{1}{e}\right] - \left[-\frac{\ln(1)}{1} - 1\right]$$

$$= \frac{-2}{e} + 1 = 1 - \frac{2}{e} = \frac{e-2}{e}$$

$$\int_1^4 e^{\sqrt{x}}\,dx = \int_1^2 e^u \cdot 2\sqrt{x}\,du \Rightarrow \int_1^2 2ue^u\,du = 2ue^u - \int 2e^u\,du =$$

$u = \sqrt{x}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$dx = 2\sqrt{x}\,du$

$x=1,\ u=1$

$x=4,\ u=2$

$w = 2u$ $dv = e^u\,du$

$dw = 2\,du$ $v = e^u$

$$2ue^u - 2e^u\Big|_1^2 = \left[4e^2 - 2e^2\right] - \left[2e - 2e\right]$$

$$= \boxed{2e^2}$$

Weird u-sub ⇒ I.B.P.

$$\int_0^1 4x^7 e^{x^4}\,dx = \int_0^1 4x^7 e^u \cdot \frac{du}{4x^3} = \int_0^1 x^4 e^u\,du = \int_0^1 ue^u\,du = \dots$$

$u = x^4$

$du = 4x^3\,dx$

$dx = \frac{du}{4x^3}$

$x=0,\ u=0$

$x=1,\ u=1$

$$= ue^u - e^u\Big|_0^1 =$$

$$= [e - e] - [0 - e^0] = 1$$

$$\int \ln(x)\,dx = x\ln(x) - \int x\cdot\frac{1}{x}\,dx = x\ln(x) - \int 1\,dx = \boxed{x\ln(x) - x + C}$$

$u = \ln(x) \quad dv = 1\,dx$

$du = \frac{1}{x}\,dx \quad v = x$

$$\int \tan^{-1}(x)\,dx = x\tan^{-1}(x) - \int \frac{x}{1+x^2}\,dx$$

$u = \tan^{-1}(x) \quad dv = dx$

$du = \frac{1}{1+x^2} \quad v = x$

$u = 1+x^2$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$$= x\tan^{-1}(x) - \int \frac{x}{u}\cdot\frac{du}{2x} = x\tan^{-1}(x) - \frac{1}{2}\int \frac{1}{u}\,du$$

$$= x\tan^{-1}(x) - \frac{1}{2}\ln|u| + C$$

$$= x\tan^{-1}(x) - \frac{1}{2}\ln|1+x^2| + C$$

$$= x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2) + C$$

$$= \boxed{x\tan^{-1}(x) - \ln\sqrt{1+x^2} + C}$$

Very simple integrands, $dv = dx$

$$\int e^x \sin(2x)\,dx = e^x\sin(2x) - \int 2e^x\cos(2x)\,dx$$

$u = \sin(2x)$ $dv = e^x dx$

$du = \cos(2x)\cdot 2\,dx$ $v = e^x$

$u = 2\cos(2x)$ $dv = e^x dx$

$du = -4\sin(2x)\,dx$ $v = e^x$

$$\therefore \int e^x\sin(2x)\,dx = e^x\sin(2x) - \left[2e^x\cos(2x) + \int 4e^x\sin(2x)\,dx\right]$$

$$\int e^x\sin(2x)\,dx = e^x\sin(2x) - 2e^x\cos(2x) - 4\int e^x\sin(2x)\,dx$$

$$\int e^x\sin(2x)\,dx = \frac{e^x\sin(2x) - 2e^x\cos(2x)}{5} + C$$

IBP x2, solve for the integral.

TRIG INTEGRATION

$$\sin^2(x) + \cos^2(x) = 1$$

$\div \cos^2(x)$

$\sin^2(x) = 1 - \cos^2(x)$

$\cos^2(x) = 1 - \sin^2(x)$

$\tan^2(x) + 1 = \sec^2(x)$

$\tan^2(x) = \sec^2(x) - 1$

$$\int \sin^2(x)\cos^3(x)\,dx = \int \cos(x)\cdot\left[\sin^2(x)\cdot\cos^2(x)\right]dx$$

$$= \int \cos(x)\left[\sin^2(x)\left(1-\sin^2(x)\right)\right]dx = \int u^2 - u^4\,du = \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$$

$u = \sin(x)$ $dx = \frac{du}{\cos(x)}$

$du = \cos(x)\,dx$

$$= \boxed{\frac{1}{3}\sin^3(x) - \frac{1}{5}\sin^5(x) + C}$$