01/30/2020

Last Time: Trig Substitution

Today: Partial Fractions

Future: LMs, HW

$$\int_0^2 \frac{x}{(x^2+4)^{3/2}}\,dx$$

$$\int \frac{1}{\sqrt{6-(x-1)^2}}\,dx$$

$$\int_0^2 \frac{x}{(x^2+4)^{3/2}}\,dx = \int_0^{\pi/4} \frac{2\tan\theta \cdot 2\sec^2\theta}{(4\cdot\sec^2\theta)^{3/2}}\,d\theta = \int_0^{\pi/4} \frac{4\tan\theta\sec^2\theta\,d\theta}{8\sec^3\theta}$$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

$x=0 \Rightarrow \theta = 0$

$x=2 \Rightarrow \theta = \pi/4$

$u = x^2+4$

$$= \frac{1}{2}\int_0^{\pi/4} \frac{\tan\theta}{\sec\theta}\,d\theta = \frac{1}{2}\int_0^{\pi/4} \sin\theta\,d\theta = -\frac{1}{2}\cos\theta\Big|_0^{\pi/4}$$

$$= -\frac{1}{2}\left[\cos(\pi/4) - \cos(0)\right] = -\frac{1}{2}\left[\frac{\sqrt{2}}{2} - 1\right]$$

$$= \frac{1}{2}\left[1 - \frac{\sqrt{2}}{2}\right] = \frac{1}{2}\left[\frac{2-\sqrt{2}}{2}\right] = \frac{1}{4}(2-\sqrt{2})$$

$$\int \frac{1}{\sqrt{6-(x-1)^2}}\,dx = \int \frac{\sqrt{6}\cos\theta\,d\theta}{\sqrt{6\cos^2\theta}} = \int 1\,d\theta = \theta + C$$

$x-1 = \sqrt{6}\sin\theta$

$dx = \sqrt{6}\cos\theta\,d\theta$

$\frac{x-1}{\sqrt{6}} = \sin\theta$, $\theta = \sin^{-1}\left(\frac{x-1}{\sqrt{6}}\right)$

$$\boxed{\sin^{-1}\left(\frac{x-1}{\sqrt{6}}\right) + C}$$

$$\int \frac{1}{1+x^2}\,dx = \dots = \tan^{-1}(\theta) + C$$

$x = \tan\theta$

$\vdots$

$$\int \frac{27}{x^2+3x-18}\,dx = \int \frac{27}{(x-3)(x+6)}\,dx = \cdots\cdots = \int \frac{A}{x-3} + \frac{B}{x+6}\,dx$$

Partial Fraction Decomposition.

Find a common denominator

$$\frac{27}{(x-3)(x+6)} = \frac{A}{x-3} + \frac{B}{x+6} = \frac{A(x+6)+B(x-3)}{(x-3)(x+6)}$$

$$27 = A(x+6) + B(x-3)$$

$x=-6 \quad 27 = A(0) + B(-9) \quad \therefore B = -3$

$x=3 \quad 27 = A(9) + B(0) \quad \therefore A = 3$

$$\int \frac{27}{(x-3)(x+6)}\,dx = \int \frac{3}{x-3} + \frac{-3}{x+6}\,dx$$

$$= 3\ln|x-3| - 3\ln|x+6| + C$$

$$= 3\left[\ln|x-3| - \ln|x+6|\right] + C$$

$$= 3\ln\left|\frac{x-3}{x+6}\right| + C$$

$$\int_0^1 \frac{2x-7}{x^2-x-2}\,dx$$

$$\frac{2x-7}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} = \frac{A(x+1)+B(x-2)}{(x-2)(x+1)}$$

$$2x-7 = A(x+1)+B(x-2)$$

$x=-1 \Rightarrow \quad -9 = 0 - 3B, \quad B=3$

$x=2 \Rightarrow \quad -3 = 3A + 0, \quad A=-1$

$$\ln\left|\frac{(x+1)^3}{x-2}\right|$$

$$\therefore \int_0^1 \frac{-1}{x-2} + \frac{3}{x+1}\,dx = -\ln|x-2| + 3\ln|x+1| \Big|_0^1 =$$

$$\Big[\cancel{-\ln|-1|}^{\,0} + 3\ln|2|\Big] - \Big[-\ln|-2| + \cancel{3\ln|1|}^{\,0}\Big]$$

$$3\ln 2 + \ln 2$$

$$= 4\ln(2)$$

$$= \ln(16)$$

$$= 2\ln(4)$$

$$= \tfrac{1}{2}\ln(256)$$

$$\int \frac{2x-3}{x^3+x}\,dx = \int \frac{2x-3}{x(x^2+1)}\,dx$$

$x^2+1 \Rightarrow$ Irred. Quad.

$$\frac{2x-3}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)x}{x(x^2+1)}$$

$$2x-3 = A(x^2+1)+(Bx+C)x$$

$x=0 \Rightarrow -3 = A+0 \quad \therefore \boxed{A=-3}$

$$2x-3 = (-3)(x^2+1)+(Bx+C)x$$

$$2x-\cancel{3} = -3x^2-\cancel{3}+Bx^2+Cx$$

$$\boxed{3x^2}+\boxed{2x} = \boxed{Bx^2}+\boxed{Cx}$$

$B=3$

$C=2$

$$\therefore \int \frac{2x-3}{x^3+x}\,dx = \int \frac{-3}{x} + \frac{3x+2}{x^2+1}\,dx = \int \frac{-3}{x}\,dx + \int \frac{3x}{x^2+1}\,dx + \int \frac{2}{x^2+1}$$

$-3\ln|x|$

$u = x^2+1$

$x = \tan\theta$

OR

$2\tan^{-1}(x)$

$$\int \frac{20}{x^3 - 4x^2 + 4x}\,dx = \int \frac{20}{x(x^2-4x+4)}\,dx =$$

$$\int \frac{20}{x(x-2)^2}\,dx$$

x^2+1, irred Quad

$(x-2)^2$, rep. linear

$$\frac{20}{x(x-2)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2} = \frac{A(x-2)^2 + B(x-2)\cdot x + Cx}{x(x-2)^2}$$

$$20 = A(x-2)^2 + B(x-2)\cdot x + Cx$$

$$x=0 \Rightarrow 20 = 4A + 0 + 0 \Rightarrow A = 5 \leftarrow$$

$$x=2 \Rightarrow 20 = 0 + 0 + 2C \Rightarrow C = 10 \leftarrow$$

$$20 = 5(x-2)^2 + Bx(x-2) + 10x$$

$$20 = 5(x^2-4x+4) + Bx^2 - 2Bx + 10x$$

$$\cancel{20} = 5x^2 - 20x + \cancel{20} + Bx^2 - 2Bx + 10x$$

$$\boxed{-5x^2} + \boxed{10x} = \boxed{Bx^2}\boxed{-2Bx}$$

$\therefore B = -5$

$-2B = 10 \quad \therefore B = -5$

$$\int \frac{5}{x} + \frac{-5}{x-2} + \frac{10}{(x-2)^2}\,dx$$

$$\int \frac{x^2+2}{x^2-1}\,dx = \int \frac{(x^2-1)+3}{x^2-1}\,dx = \int 1 + \frac{3}{(x+1)(x-1)}\,dx$$

$$\begin{array}{r|l} & 1 + \frac{3}{x^2-1} \\ \hline x^2-1 & x^2+2 \\ & -x^2+1 \\ \hline & 3 \end{array}$$

$$= x + \int \frac{A}{x+1} + \frac{B}{x-1}\,dx$$

$$\int \frac{2x^2+4x+1}{x^2-\cancel{2}4}\,dx = \int \frac{2(x^2-4)+4x+9}{x^2-4}\,dx = \int 2 + \frac{4x+9}{x^2-4}\,dx$$

$$\begin{array}{r|l} & 2 + \frac{4x+9}{x^2-4} \\ \hline x^2-4 & 2x^2+4x+1 \\ & -2x^2 \quad\; +8 \\ \hline & 4x+9 \end{array}$$