02/04/2020

Last Time: Partial Fractions

Today: Improper Integrals

Future: LMs, HW, Exam I

$$\int_1^5 \frac{10}{x^3+x^2}\,dx$$

u-sub ×
IBP ×
Trig ×
Trig sub ×
PF ✓

x^2 ~~irred Quad~~
⇒ rep Lin.

$$\frac{10}{x^2(x+1)} = \frac{A}{x+1} + \frac{B}{x} + \frac{C}{x^2} = \frac{Ax^2 + Bx(x+1) + C(x+1)}{x^2(x+1)}$$

$$\therefore \quad 10 = Ax^2 + Bx(x+1) + C(x+1)$$

$$x=0 \Rightarrow 10 = 0 + 0 + C \Rightarrow C = 10.$$

$$x=-1 \Rightarrow 10 = A + 0 + 0 \Rightarrow A = 10.$$

$$10 = 10x^2 + Bx^2 + Bx + 10x + 10$$

$$\underbrace{-10x^2}\underbrace{-10x} = \underbrace{Bx^2} + \underbrace{Bx}$$

$$\therefore \quad B = -10$$

$$\therefore \int_1^5 \frac{10}{x+1} + \frac{-10}{x} + \frac{10}{x^2}\,dx = 10\ln(x+1) - 10\ln(x) - \frac{10}{x}\Big|_1^5$$

$$= \left[10\ln(6) - 10\ln(5) - \frac{10}{5}\right] - \left[10\ln(2) - \cancel{10\ln(1)}^{\,0} - 10\right]$$

$$= 10\ln\left(\frac{3}{5}\right) + 8$$

$$\frac{1}{(x+2)^3 (x^3+5)\cdot(x^2+2)^2\cdot x} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3} +$$

$$+ \frac{Dx^2+Ex+F}{x^3+5} + \frac{Gx+H}{x^2+2} + \frac{Ix+J}{(x^2+2)^2} + \frac{K}{x}$$

Find P.F. Decomposition:

$$\frac{12x}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} = \frac{A(x^2+x+1)+(x-1)(Bx+C)}{(x-1)(x^2+x+1)}$$

$$\Rightarrow 12x = A(x^2+x+1) + (x-1)(Bx+C)$$

$x=1 \Rightarrow \quad 12 = 3A + 0 \quad \therefore A=4$

$$12x = 4(x^2+x+1) + Bx^2+Cx-Bx-C$$

$$8x = 4x^2+4+Bx^2+Cx-Bx-C$$

$$\underbrace{-4x^2} + \underbrace{8x} \underbrace{-4} = \underbrace{Bx^2} + \underbrace{Cx-Bx} \underbrace{-C}$$

$\therefore B=-4$

$C=4$

$8 = C-B = 4--4 = 8$

$$\therefore \frac{4}{x-1} + \frac{-4x+4}{x^2+x+1}$$

$$\int_0^1 \frac{t}{\sqrt{2-t^4}}\,dt = \int_0^1 \frac{t}{\sqrt{(\sqrt{2})^2-(t^2)^2}}\,dt = \int_0^{\pi/4} \frac{t}{\sqrt{2-2\sin^2\theta}} \cdot \frac{\sqrt{2}\cos\theta}{2t}$$

$t^2 = \sqrt{2}\sin\theta$

$2t\,dt = \sqrt{2}\cos\theta\,d\theta$

$t=0 \Rightarrow \theta=0$

$t=1 \Rightarrow \theta=\pi/4$

$$= \frac{1}{2}\int_0^{\pi/4} \frac{\sqrt{2}\cos\theta\,d\theta}{\sqrt{2}\cdot\cos\theta} = \frac{1}{2}\int_0^{\pi/4} d\theta$$

$$= \frac{\theta}{2}\Big|_0^{\pi/4} = \boxed{\pi/8}$$

$$\int_4^8 \frac{2\sqrt{x-4}}{x}\,dx = \int_0^2 \frac{2u}{x}\cdot 2\sqrt{x-4}\,du = \int_0^2 \frac{4u^2}{x}\,du$$

$u = \sqrt{x-4} \Leftrightarrow u^2 = x-4$

$\therefore x = u^2+4$

$$= \int_0^2 \frac{4u^2}{u^2+4}\,du$$

$du = \frac{1}{2}(x-4)^{-1/2}dx$

$= \frac{1}{2\sqrt{x-4}}\,dx$

$dx = 2\cdot\sqrt{x-4}\,du$

$x=4,\ u=0$

$x=8,\ u=2$

$u = 2\tan\theta$

$\vdots$

An Integral is <u>Improper</u> if there is an ∞ somewhere in the integral

$$\int_1^{\infty} \frac{1}{x+3}\,dx, \quad \int_{-\infty}^{\infty} e^{-1/x^2+1}\,dx, \quad \int_{-2}^{2} \frac{1}{x}\,dx, \quad \int_1^{5} \frac{4}{\sqrt{5-x}}\,dx$$

$$\int_1^{4} \frac{5}{\sqrt{5-x}}\,dx$$

Def^n: If we integrate an improper integral and evaluate and get one, finite #, we say the integral <u>converges</u> to that number.

Otherwise, we say it <u>diverges</u>

Converges $\Longleftrightarrow$ one finite number

Diverges $\Longleftrightarrow$ · ∞ or $-\infty$
· oscillates

$$\int_1^{\infty} \frac{1}{x}\,dx = \ln(x)\Big|_1^{\infty} = \ln(\infty) - \cancel{\ln(1)}^{0}$$

$$= \infty$$

$\therefore$ Diverges

$$\int_0^{\infty} \cos(x)\,dx =$$

$$\sin(x)\Big|_0^{\infty} = \sin(\infty) - \sin(0)$$

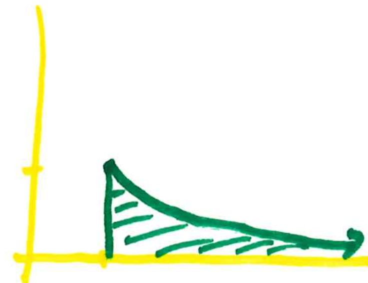
$$= \sin(\infty)$$

$\therefore$ Diverges.

Proper integrals always converge

Improper integral converge or diverge.

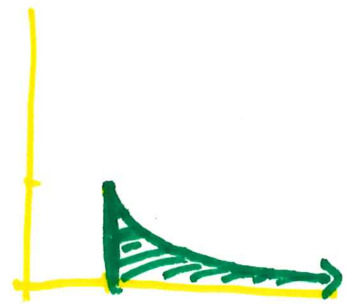
$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \int_1^\infty x^{-1/2}\,dx = 2\sqrt{x}\,\Big|_1^\infty$$



$$= 2\sqrt{\infty} - 2\sqrt{1}$$

$$= \infty \text{ , Div.}$$

$$\int_1^\infty \frac{1}{x^2}\,dx = \int_1^\infty x^{-2}\,dx = -x^{-1}\Big|_1^\infty$$



$$= -\frac{1}{\infty} - \frac{-1}{1} =$$

$$= 1$$

$$\int_0^1 \frac{4}{\sqrt[3]{1-x}}\,dx = \int_1^0 \frac{4}{\sqrt[3]{u}} \cdot -du = \int_0^1 4u^{-1/3}\,du = \frac{4u^{2/3}}{2/3}\Big|_0^1$$

$u = 1-x$

$du = -dx$

$x = 1,\ u = 0$

$x = 0,\ u = 1$

$$= \frac{3}{2} \cdot 4 \cdot u^{2/3}\Big|_0^1 = 6 - 0 = 6$$

$$\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = \int_1^\infty \frac{e^{-u}}{\sqrt{x}} \cdot 2\sqrt{x}\,du = 2\int_1^\infty e^{-u}\,du = -2e^{-u}\Big|_1^\infty$$

$u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}dx$

$dx = 2\sqrt{x}\,du$

$x = 1,\ u = 1$

$x = \infty,\ u = \infty$

$$= -2e^{-\infty} - -2e^{-1}$$

$$= \frac{-2}{e^{\infty}} + \frac{2}{e} = \frac{2}{e}$$

$=$

$$\int_{-2}^{2} \frac{1}{x^2}\,dx = \cancel{-x^{-1}\Big|_{-2}^{2} = -\frac{1}{2} - \frac{-1}{-2} = -\frac{1}{2} - \frac{1}{2} = \boxed{-1}}$$

Is All wrong.

$$= \underbrace{\int_{-2}^{0} x^{-2}\,dx} + \int_{0}^{2} x^{-2}\,dx$$

$$\int_{-2}^{0} x^{-2}\,dx = -x^{-1}\Big|_{-2}^{0} = -\frac{1}{0} - \frac{-1}{-2} \Rightarrow \boxed{\text{Diverges.}}$$