02/06/2020

Last Time: Improper Integrals §7.8

Today: Strategies for Integration §7.5

Future: LMs, HW, Exam 1

Exam I

- This classroom
- 75 Minutes
- No Notes, No Calculators
- 12 M.C. Qns, 8 pts each → 96
- 3 T/F Qns, 3 pts each → 9

105 possible

I will bring formula sheet + scratch paper

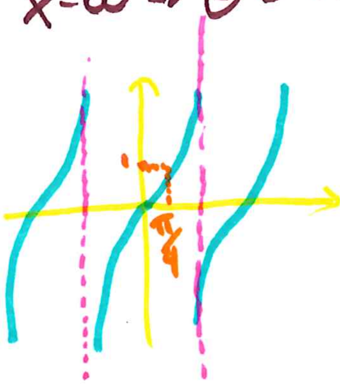
- Backpacks go in front or on side.
- Cellphones off
- Sit as close to the front as possible
- Fill in each seat of the row.

$$\int_1^{\infty} \frac{2}{x^2\sqrt{x^2+1}}\,dx = \int_{\pi/4}^{\pi/2} \frac{2}{\tan^2\theta \cdot \sec\theta}\sec^2\theta\, d\theta = \int_{\pi/4}^{\pi/2} \frac{2\sec\theta}{\tan^2\theta}\, d\theta =$$

$$\begin{cases} x = \tan\theta \\ dx = \sec^2\theta\, d\theta \end{cases}$$

$x = 1 \Rightarrow \theta = \pi/4$

$x = \infty \Rightarrow \theta = \pi/2$



$$= \int_{\pi/4}^{\pi/2} \frac{2 \cdot \frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\, d\theta = \int_{\pi/4}^{\pi/2} 2\frac{1}{\cos\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta}\, d\theta$$

$$= 2\int_{\pi/4}^{\pi/2} \frac{\cos\theta}{\sin^2\theta}\, d\theta = 2\int_{\sqrt{2}/2}^{1} u^{-2}\, du = \frac{-2}{u}\Big|_{\frac{\sqrt{2}}{2}}^{1}$$

$u = \sin\theta$

$du = +\cos\theta$

$\theta = \pi/2,\ u = 1$

$\theta = \pi/4,\ u = \frac{\sqrt{2}}{2}$

$$= -\frac{2}{1} - \frac{-2}{\frac{\sqrt{2}}{2}} =$$

$$= -2 + \frac{4}{\sqrt{2}} =$$

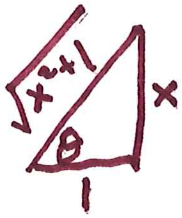
$$= -2 + \frac{4\sqrt{2}}{2} =$$

$$= \boxed{2\sqrt{2} - 2 = 2(\sqrt{2} - 1)}$$

Suppose we didn't switch or limits of integration, or this was indefinite:

$$\therefore \frac{-2}{u} = \frac{-2}{\sin\theta} = \frac{-2\sqrt{x^2+1}}{x}\Bigg|_1^{\infty}$$

$\tan\theta = \frac{x}{1} = \frac{opp}{adj}$



$\therefore \sin\theta = \frac{opp}{hyp} = \frac{x}{\sqrt{x^2+1}}$

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^3}\,dx$$

- ~~Basic Rule~~
- u-sub
- ~~IBP~~
- ~~Trig~~
- ~~Trig sub~~
- ~~P.F.~~

$$u = \frac{1}{x}$$

$$du = -\frac{1}{x^2}\,dx$$

$$dx = -x^2\,du$$

$$\Rightarrow \int \frac{\cos(u)}{x^3}\cdot -x^2\,du = -\int \frac{\cos(u)}{x}\,du = -\int u\cos(u)\,du$$

$w = u \qquad dv = \cos(u)\,du$

$dw = du \qquad v = \sin(u)$

$$= -\left[u\sin(u) - \int \sin(u)\,du\right]$$

$$= -u\sin(u) + \int \sin(u)\,du$$

$$= -u\sin(u) - \cos(u) + C$$

$$= \boxed{\frac{-1}{x}\sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) + C}$$

$$\int_0^1 \frac{1}{1+e^x}\,dx$$

- ~~Basic~~
- u-sub
- ~~IBP~~
- ~~Trig~~
- Trig sub
- ~~PF~~

$$\int_0^1 \frac{1}{1+(e^{x/2})^2}\,dx$$

$\therefore e^{x/2} = \tan\theta$

This works too.

$u = e^x$

$du = e^x dx$

$dx = \frac{du}{e^x}$

$u = e^x + 1$

$x=0,\ u=1$

$x=1,\ u=e$

$$\therefore \int_1^e \frac{1}{1+u}\cdot\frac{du}{e^x} = \int_1^e \frac{du}{(u+1)\cdot u} \overset{P.F}{=\cdots\cdots=} \int_1^e \frac{-1}{u+1} + \frac{1}{u}\,du$$

$$\therefore -\ln|u+1| + \ln|u|\Big|_1^e = \Big[-\ln(e+1)+\ln(e)\Big] - \Big[-\ln(2)+\ln(1)\Big]$$

$$= -\ln(e+1) + 1 + \ln(2)$$

$$= \boxed{1 + \ln\left(\frac{2}{e+1}\right)}$$

$$\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx = \int_0^{\pi/4} \frac{\sin^2\theta}{\cos\theta}\cos\theta\,d\theta = \int_0^{\pi/4} \sin^2\theta\,d\theta = \int_0^{\pi/4} \frac{1}{2} - \frac{\cos(2\theta)}{2}$$

$x = \sin\theta$

$dx = \cos\theta\,d\theta$

$x=0,\ \theta=0$

$x=\frac{\sqrt{2}}{2},\ \theta=\pi/4$

$$= \frac{\theta}{2} - \frac{\sin 2\theta}{4}\Big|_0^{\pi/4}$$

$$= \left[\frac{\pi}{8} - \frac{\sin(\pi/2)}{4}\right] - [0-0] =$$

$$\frac{\pi}{8} - \frac{1}{4} = \frac{1}{4}\left[\frac{\pi}{2} - 1\right] =$$

$$= \frac{1}{8}[\pi - 2]$$

$$\int_0^1 \frac{\sin^{-1}(\sqrt{x})}{\sqrt{x}}\,dx = \int_0^1 \frac{\sin^{-1}(u)}{\sqrt{x}} \cdot 2\sqrt{x}\,du$$

$$= 2\int_0^1 \sin^{-1}(u)\,du = 2\left[u\sin^{-1}(u) - \int \frac{u}{\sqrt{1-u^2}}\,du\right]$$

$u = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$dx = 2\sqrt{x}\,du$

$x=0,\ u=0$

$x=1,\ u=1$

$u = \sin^{-1}(u)$ $\quad dv = du$

$du = \frac{1}{\sqrt{1-u^2}}$ $\quad v = u$

$w = 1-u^2$

$dw = -2u\,du$

$$\ldots\ 2u\sin^{-1}(u) + \int w^{-1/2}\,dw$$

$$= 2u\sin^{-1}(u) + 2\sqrt{w} = 2u\sin^{-1}(u) + 2\sqrt{1-u^2}\Big|_0^1$$

$$= \ldots = \pi - 2$$