02/18/2020

Last Time: Intro to D.E.

Today: Slope Fields + Euler's Method

Future: LMs + HW05

① Show that $y = \frac{1}{(C+x^2)^2}$ is a sol'n to $y' = -4xy^{3/2}$

② If $y' = -4xy^{3/2}$ and $y(1) = 1/4$, find y.

$$\left[(C+x^2)^{-2}\right]' \stackrel{?}{=} -4x\left((C+x^2)^{-2}\right)^{3/2}$$

$$-2(C+x^2)^{-3} \cdot 2x \stackrel{?}{=} -4x(C+x^2)^{-3}$$

$$\frac{-4x}{(C+x^2)^3} = \frac{-4x}{(C+x^2)^3} \checkmark$$

$$y(x) = \frac{1}{(C+x^2)^2} \text{ AND } y(1) = 1/4 \Rightarrow \frac{1}{(C+1^2)^2} = \frac{1}{4} \Rightarrow$$

$$\Rightarrow (C+1)^2 = 4 \Rightarrow C+1 = \pm 2 \quad \therefore C = \pm 2 - 1$$

$$= -3, 1$$

$$\therefore y(x) = \frac{1}{(1+x^2)^2} \text{ or } \frac{1}{(-3+x^2)^2}$$

What we know about D.Eqns.

① Given y, and a D.E., we can check if y satisfies the D.E.

② Given y and a point $y(x_0) = y_0$, Find The particular solution thru that point (I.V.P.)

Our Goal is to find a function y which is a soln to a DE... This is often really, REALLY, REALLY tough.

① Sometimes we will find y.

② Sometimes we will not be able to find y, but other people can.

③ Sometimes there are problems where NO ONE, currently, can solve it.

Today we will not make a distinction between these 3 categories. Instead we will use techniques that apply to all D. Eqns.

Suppose $y' = x - y$. Can we find y? Yes, next week.

What can we say about y without knowing exactly what it is.

It is <u>super</u> easy to find y' at a point

$(0,2) \Rightarrow y' = 0 - 2 = -2$

$(0,1) \Rightarrow y' = -1$

$(0,0) \Rightarrow y' = 0$

$y(0,-1) \Rightarrow y' = 1$

$y(0,-2) \Rightarrow y' = 2$

$y(1,2) \Rightarrow y' = -1$

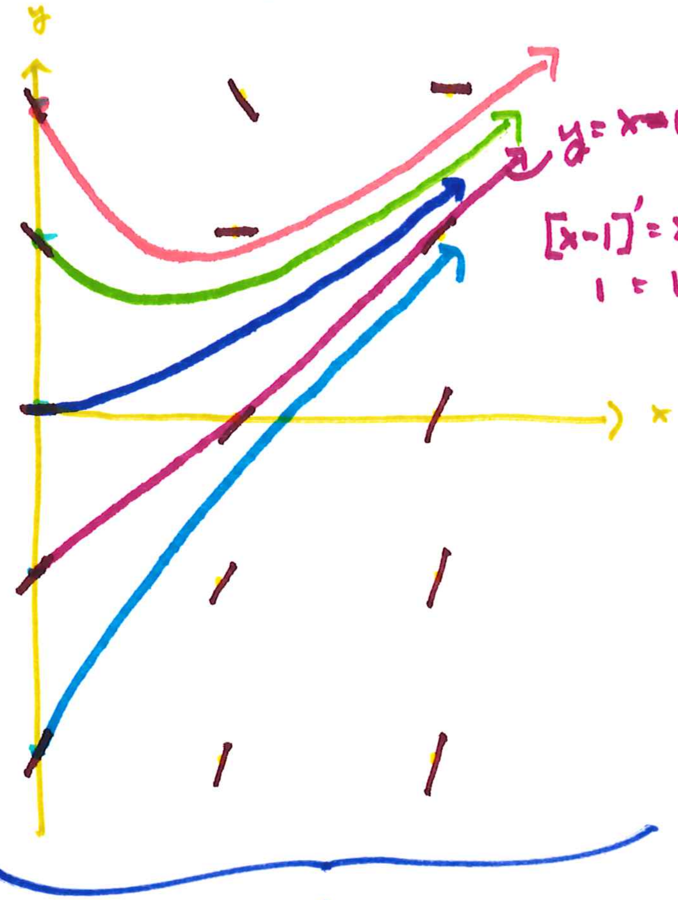
$y(1,1) \Rightarrow y' = 0$

$\vdots \quad \vdots \quad \vdots$

$y(2,2) \Rightarrow y' = 0$

$\vdots \quad \vdots$

$y(2,-2) \Rightarrow y' = 4$



$[x-1]' = x - (x-1)$

$1 = 1$ ✓

Slope Field

Directional Field.

Claim: $y = x - 1 + Ce^{-x}$ is the soln to $y' = x - y$

$$[x-1+Ce^{-x}]' = x - (x - 1 + Ce^{-x})$$

$$1 - Ce^{-x} = 1 - Ce^{-x} \checkmark$$

Let $y' = y^2 - 4$, find $\lim\limits_{x\to\infty} y$

$y=3,\ y'=5$

$y=2,\ y'=0$

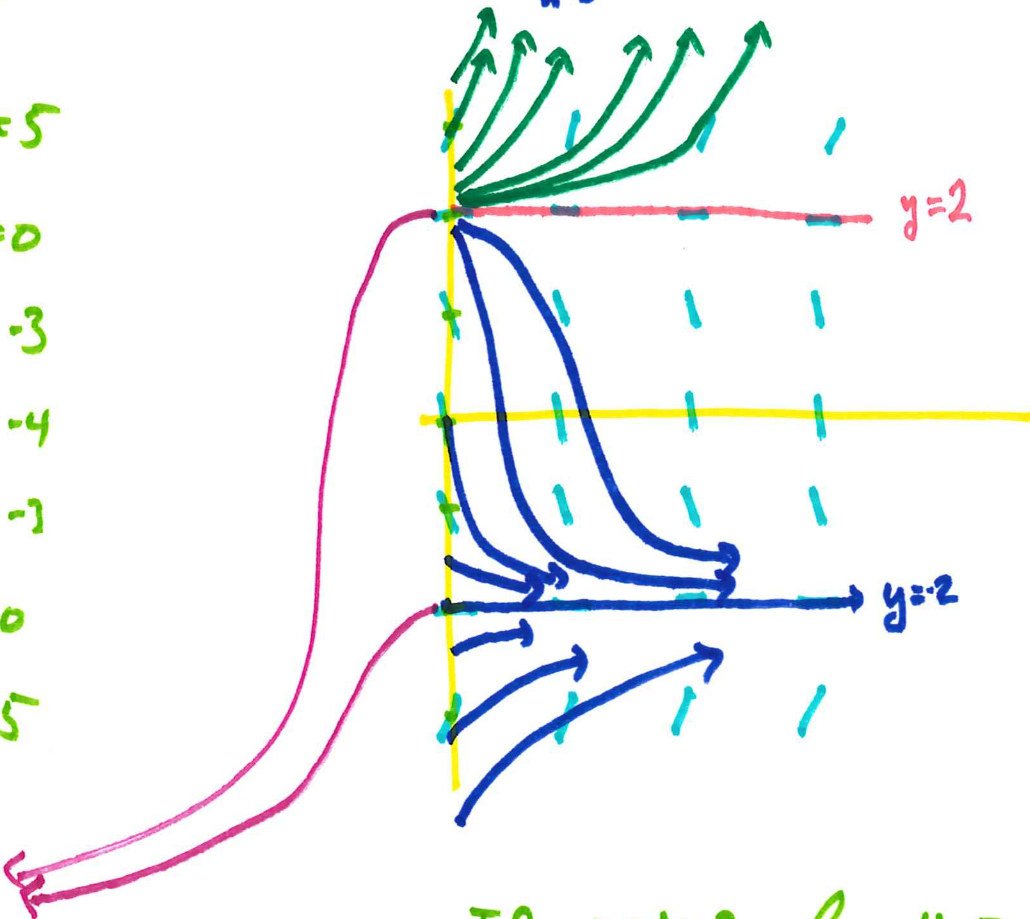
$y=1,\ y'=-3$

$y=0,\ y'=-4$

$y=-1,\ y'=-3$

$y=-2,\ y'=0$

$y=-3,\ y'=5$



Equilibrium Solutions, $y'=0$

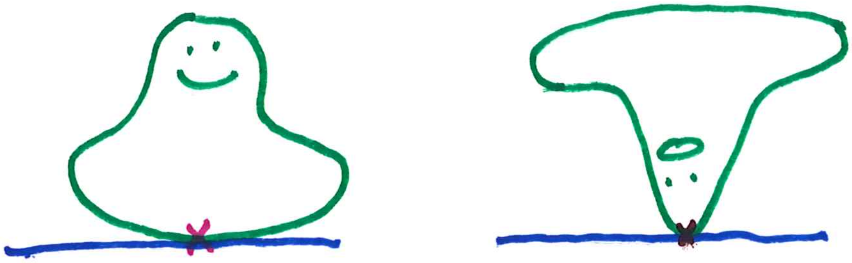
If $y(0)>2$, $\lim\limits_{x\to\infty} y = \infty$

If $y(0)<2$, $\lim\limits_{x\to\infty} y = -2$

If $y(0)=2$, $\lim\limits_{x\to\infty} y = 2$

$y=2$ is unstable equilibrium

$y=-2$ is stable equilibrium

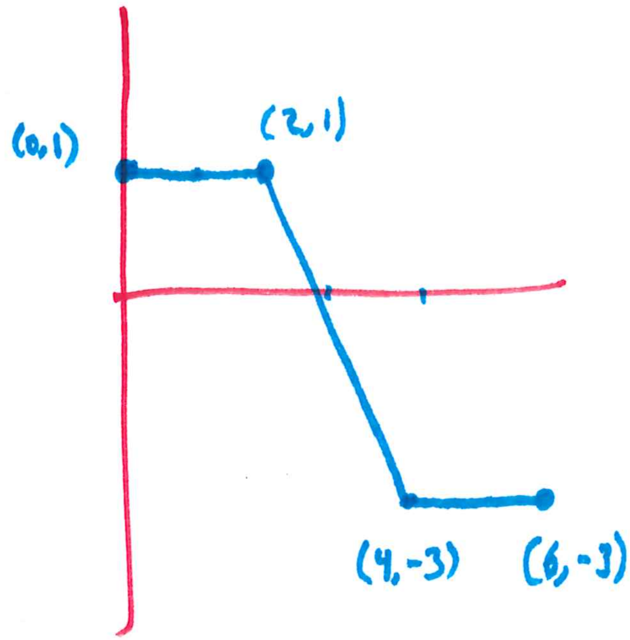


"gömböc"

a 3D thing with exactly one stable equilibrium.

Suppose $y' = 1 - x - y$ and $y(0) = 1$. Approximate $y(6)$.

I am going to do this using 3 steps, where $\Delta x = 2$



$(0,1) \Rightarrow y' = 1 - 0 - 1 = 0$

$(2,1) \Rightarrow y' = 1 - 2 - 1 = -2$

$(4,-3) \Rightarrow y' = 1 - 4 + 3 = 0$

Euler's Method.

$y(6) \approx -3$

Let $y' = 2 + 2x - 4y$, $y(0) = 1$, $\Delta x = h = \frac{1}{2}$. Approx $y(2)$.

$y_{n+1} = h \cdot y' + y_n$

- $\rightarrow$ y' at (x_n, y_n)
- $\rightarrow$ h step size, Δx
- $\rightarrow$ y_{n+1} is next y-value

$(x_0, y_0) = (0, 1)$ $\quad y_1 = h \cdot y' + y_0 = (\frac{1}{2})(2 + 0 - 4) + 1 = 0$

$(x_1, y_1) = (\frac{1}{2}, 0)$ $\quad y_2 = h \cdot y' + y_1 = (\frac{1}{2})(2 + 1 - 0) + 0 = \frac{3}{2}$

$(x_2, y_2) = (1, \frac{3}{2})$ $\quad y_3 = h \cdot y' + y_2 = (\frac{1}{2})(2 + 2 - 6) + \frac{3}{2} = \frac{1}{2}$

$(x_3, y_3) = (\frac{3}{2}, \frac{1}{2})$ $\quad y_4 = h \cdot y + y_3 = (\frac{1}{2})(2 + 3 - 2) + \frac{1}{2} = 2$

$(x_4, y_4) = (2, 2)$

$\therefore y(2) \approx 2$